

# Mathematics - III (Teaching Mathematics Pedagogy Option)

WINDOWS ON PRACTICE GUIDE  
B.Ed. (Hons.) Elementary

2012



This product has been made possible by the support of the American People through the United States Agency for International Development (USAID). The contents of this report are the sole responsibility of the authors, and do not necessarily reflect the views of USAID or the United States Government.

**Technical Support:** Education Development Center (EDC); Teachers College, Columbia University



## Higher Education Commission

### Foreword

Teacher education in Pakistan is leaping into the future. This updated Scheme of Studies is the latest milestone in a journey that began in earnest in 2006 with the development of a National Curriculum, which was later augmented by the 2008 National Professional Standards for Teachers in Pakistan and the 2010 Curriculum of Education Scheme of Studies. With these foundations in place, the Higher Education Commission (HEC) and the USAID Teacher Education Project engaged faculty across the nation to develop detailed syllabi and course guides for the four-year B.Ed. (Hons) Elementary and the two-year Associate Degree in Education (ADE).

The syllabi and course guides have been reviewed by the National Curriculum Review Committee (NCRC) and the syllabi are approved as the updated Scheme of Studies for the ADE and B.Ed. (Hons) Elementary programmes.

As an educator, I am especially inspired by the creativity and engagement of this updated Scheme of Studies. It offers the potential for a seismic change in how we educate our teachers and ultimately our country's youngsters. Colleges and universities that use programmes like these provide their students with the universally valuable tools of critical thinking, hands-on learning, and collaborative study.

I am grateful to all who have contributed to this exciting process, in particular the faculty and staff from universities, colleges, and provincial institutions who gave freely of their time and expertise for the purpose of preparing teachers with the knowledge, skills, and dispositions required for nurturing students in elementary grades. Their contributions to improving the quality of basic education in Pakistan are incalculable. I would also like to thank the distinguished NCRC members, who helped further enrich the curricula by their recommendations. The generous support received from the United States Agency for International Development (USAID) enabled HEC to draw on technical assistance and subject-matter expertise of the scholars at Education Development Center, Inc., and Teachers College, Columbia University. Together, this partnership has produced a vitally important resource for Pakistan.

PROF. DR SOHAIL NAQVI  
Executive Director  
Higher Education Commission  
Islamabad

## How this course guide was developed

As part of nationwide reforms to improve the quality of teacher education, the Higher Education Commission (HEC), with technical assistance from the USAID Teacher Education Project, engaged faculty across the nation to develop detailed syllabi for courses in the new four-year B.Ed. (Hons) Elementary programme.

The process of designing the syllabus for each course in years 3–4 of the programme began with curriculum design workshops. Deans and directors from universities where these courses will be taught were invited to attend the workshops. The first workshop included national and international subject matter experts who led participants in a seminar focused on a review and update of subject (content) knowledge. The remainder of this workshop was spent reviewing the HEC Scheme of Studies, organizing course content across the semester, developing detailed unit descriptions, and preparing the course syllabi. Although the course syllabi are designed primarily for Student Teachers taking the course, they are useful resources for teacher educators too.

Following the initial workshop, faculty participants developed teaching notes that included ideas for teaching units of study and related resources. Working individually or in groups, participants focused on their own teaching methods and strategies and how they could be useful to future teachers of the course. Subsequent workshops were held over the course of a year to give faculty sufficient time to complete their work, engage in peer review, and receive critical feedback from national and international consultants. In designing both the syllabi and the teaching notes, faculty and subject matter experts were guided by the National Professional Standards for Teachers in Pakistan (2009).

All of the syllabi developed by faculty who participated in the workshops are included in this document, along with a list of topical teaching notes. Additional references and resources appear at the end of the document. These should provide a rich resource for faculty who will teach this course in the future. Sample syllabi with accompanying teaching notes are also included to provide new Instructors with a model for developing curricula and planning to teach. This Windows on Practice guide is not intended to provide a complete curriculum with a standard syllabus and fully developed units of study, but rather aims to suggest ideas and resources for Instructors to use in their own planning. Hence, readers will find sample units and materials that reflect the perspective of faculty designers rather than prescriptions for practice.

We respect intellectual property rights and to the best of our knowledge, have not included any suggested materials that are copyright protected or for which we have not secured explicit permission to use. Therefore, all materials included may be used in classrooms for educational purposes. Materials in this document are not intended for commercial use, however. They may not be used in other publications without securing permission for their use.

Initial drafts were reviewed by the National Curriculum Review Committee (NCRC) and suggestions were incorporated into final drafts, which were then submitted to the NCRC for approval.

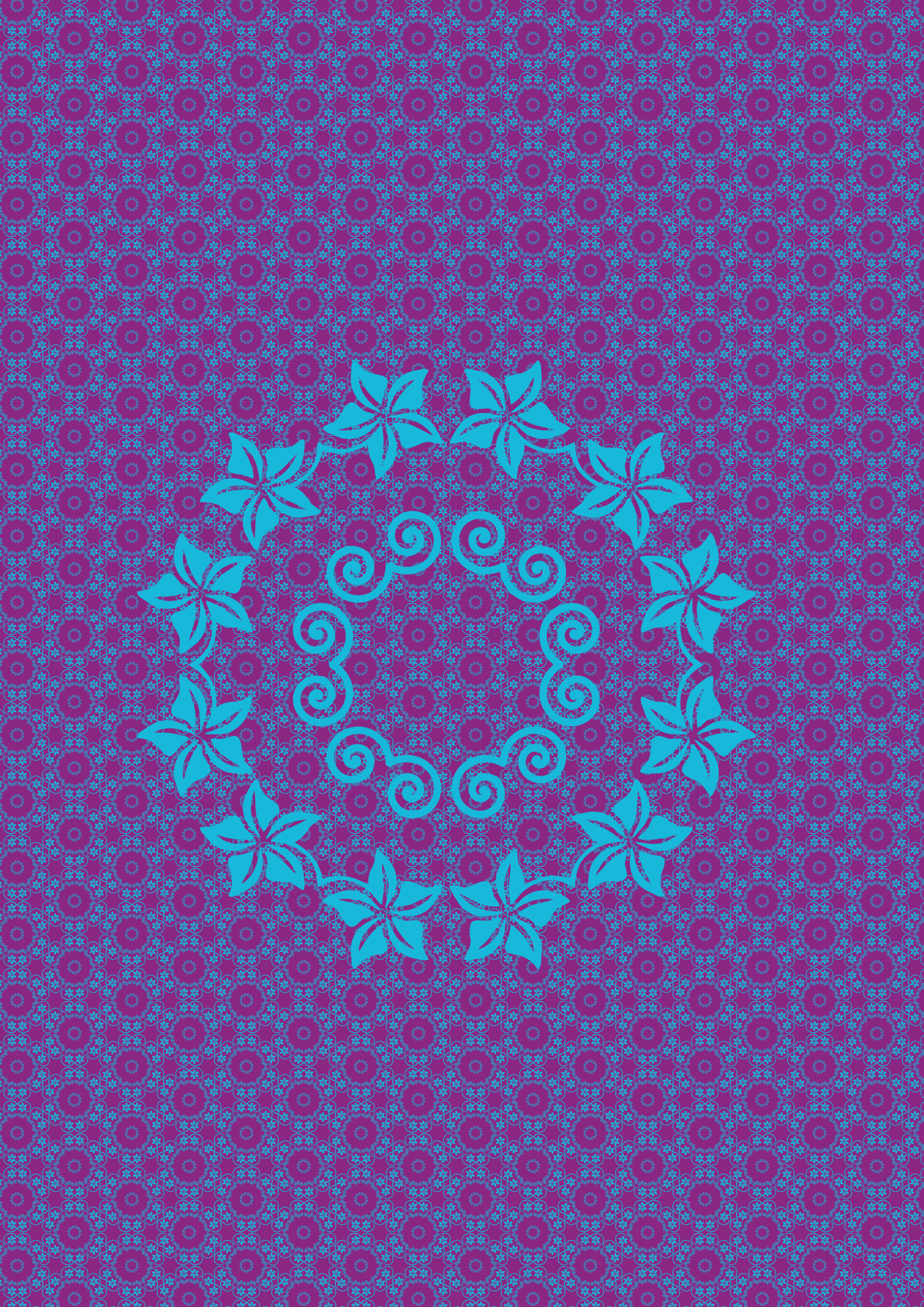
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National and international subject experts leading the seminar: Dr Munira Amirali, Mathematics Educationalist, Manager Academics, Aga Khan Education Service, Pakistan; and Loretta Heuer, Senior Research and Development Associate, Division of Mathematics, Education Development Center.

National subject expert facilitating the curriculum design process: Dr Munira Amirali, Mathematics Educationalist, Manager Academics, Aga Khan Education Service, Pakistan.

NCRC review dates: 24 & 25 April 2013

NCRC reviewers: Dr Muhammad Imran Yousuf, Arid Agricultural University, Rawalpindi; Prof. Dr Rehana Masroor, Allama Iqbal Open University, Islamabad; and Dr Riaz ul Haq Tariq, Bahauddin Zakariya University, Multan.



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# Rationale for a course on Mathematics III



In the HEC 2010 document, *Teaching of Mathematics: B.Ed (Hons.) 4 year Degree Program (Elementary & Secondary, Associate Degree in Education, M. Ed. / Ms. Education)*, Teaching of Mathematics has been included as a professional course. This course will equip Student Teachers with the knowledge and skills to teach mathematics in grades 1 to 8. They will become familiar with the National Curriculum for Mathematics (2006) and expected student learning outcomes. The National Curriculum emphasizes a problem-solving approach to teaching and learning mathematics, in which teachers are expected to promote students' active involvement in doing mathematics, rather than practising knowledge transmission from an authority (teacher) to the receiver (student). Mathematics learning can inculcate problem-solving, logical-thinking, and reasoning skills in students only when they are taught in such a way that they learn conceptually instead of by drill and practice. Therefore, this course, Mathematics III, intends to build Student Teachers' understanding of the nature of mathematics and its teaching and learning, while also creating an awareness of the history of mathematics and its scope and significance. Student Teachers will learn to use a variety of instructional methods that promote the active learning of mathematics, including making and using teaching and learning materials. They will plan mathematics lessons and activities and practice teaching mathematics with peers.

The following main ideas are discussed in this course:

- The nature and scope of mathematics
- Teachers' perceptions that influence mathematics teaching and learning
- Mathematical processes
- Planning for teaching

## Common misconceptions about mathematics education

Generally, mathematics is taught through a transmission mode of teaching, whereby mathematics teachers solve one or two problems from the prescribed textbook, explain the procedures to solve problems using a formula or rules, and then assign the exercises given in the textbook (Amirali<sup>1</sup>, 2000; Warwick & Reimers<sup>2</sup>, 1995). One possible reason is the misconceptions about mathematics teaching and learning held by the public in general and by mathematics teachers in particular. It is likely that Student Teachers will enter the B.Ed. (Hons) course with some or all of the common misconceptions listed below. The course Instructor needs to be aware of these and create ample opportunities for Student Teachers to reflect critically on their beliefs and perceptions about mathematics and its teaching and learning.

<sup>1</sup> Amirali, M. (2000). *Enabling mathematics teachers to teach for conceptual understanding*. (Unpublished master's thesis). Institute for Educational Development, Aga Khan University, Karachi.

<sup>2</sup> Warwick, D. P., & Reimers, F. (1995). *Hope or despair? Learning in Pakistan's primary schools*. Westport, CT: Praeger.

## Misconceptions

- Mathematics is an abstract subject, which makes the learner less sensitive to reality.
- One needs to memorize a lot of facts, rules, and formulae to be good at mathematics.
- There is no allowance for creativity in teaching and learning mathematics.
- Everyone cannot learn mathematics, as it requires a special brain.
- Boys are naturally better at learning mathematics than girls.
- The prescribed textbook is the sole source of knowledge in mathematics teaching.
- In mathematics, getting the right answer is very important.
- There is only one way to solve mathematical problems.
- Solving a mathematical task with different methods creates confusion among students.
- The use of a calculator and computer software in a mathematics classroom hampers students' learning.
- Research-based best practices are not applicable in the Pakistani context owing to large class sizes.
- Planning for teaching is waste of time for experienced teachers.
- Mathematics can be learnt best individually instead of learning with others.
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# Course syllabi

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MATHEMATICS III  
(TEACHING MATHEMATICS  
PEDAGOGY OPTION)

This section contains three syllabi written by groups of faculty. Using the HEC Scheme of Studies for the course, they considered the balance between the demands of the subject itself, active learning pedagogies, their students, and the particular university milieu in which they work. The syllabi all reflect the same key concepts and broad goals, but they vary in sequence and emphasis.



## SYLLABUS 1

### By

Dr Muhammad Tanveer Afzal, Dr Muhammad Rauf, and Saima Kashif

### Year and semester

Year 4, Semester 7

### Credit value

3 credit hours

### Prerequisites

Mathematics 1, Mathematics 2

## Course description

Mathematics is the mother of all subjects. It appears in all walks of life; even a mason has to calculate the area of the building when claiming his wages. But teachers' existing beliefs about and perceptions of teaching mathematics in our context are not promising. We are more focused on the transmission of knowledge by engaging students in memorizing mathematical rules and formulae, rather than on engaging them in constructing mathematical knowledge and understanding mathematical concepts. Mathematics learning can inculcate problem-solving, logical-thinking, and reasoning skills in students only when they are taught in such a way that they learn conceptually instead of by drill and practice. In previous semesters, we have focused on mathematics content, but this course intends to extend Student Teachers' understanding of pedagogy as well as build their understanding of the nature of mathematics, teacher beliefs and perceptions, and mathematics teaching and learning. This will enable Student Teachers to develop students' problem-solving, logical-thinking, and reasoning skills. This course will help in creating awareness of the history of mathematics as well as its scope and significance. Also Student Teachers will be able to design plans for integrating Information and Communications Technology (ICT) to develop students' mathematical learning. The importance of designing effective assessment items to facilitate students' learning is also considered.

The following main ideas are discussed in this course:

- The nature and scope of mathematics
- The attitude of teachers towards mathematics learning and their perception of it
- Research in mathematical processes
- Planning for assessment and teaching

## Course outcomes

After completing this course, Student Teachers will be able to:

- attain a better understanding of mathematical ideas
- revisit beliefs, ideas, and perceptions about teaching and learning mathematics
- acquire the skills and competencies required for teaching mathematics at elementary level
- effectively apply the various methods, techniques, and strategies of teaching mathematics
- appreciate mathematical processes and discover the power of mathematical thinking
- appreciate learning by doing rather than instrumental learning
- develop a positive attitude towards teaching and learning mathematics
- design a unit plan for teaching and managing a classroom effectively
- design assessments for/of/as learning to facilitate students' learning
- use ICT in teaching and learning mathematics.

## Learning and teaching approaches

The following approaches will be used in the course.

- Activity-based teaching
- Inquiry method
- Discovery method
- Exploration method
- Demonstration method
- Lecture method
- Discussion with peers and Instructor
- Use of ICTs to facilitate learning and teaching

Also refer to the link available on the HEC website:

➤ [http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20\(Presentation\)/COMMON%20TEACHING%20METHODS.pdf](http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20(Presentation)/COMMON%20TEACHING%20METHODS.pdf)

The site provides a PowerPoint presentation on the above-mentioned methods of teaching, listing their strengths and weaknesses. This will help Student Teachers explore the advantages and disadvantages of teaching methods and take informed decisions on selecting appropriate teaching methods, considering the content and concept of teaching, the learning environment, and the resources available for teaching.

# Semester outline

## Unit 1: The nature, nurture, and scope of mathematics

- The nature of and philosophical thoughts underlying mathematics
- Transmission versus construction of mathematical knowledge
- Instrumental versus relational understanding
- Discussion on the National Curriculum of Mathematics

## Unit 2: Teacher beliefs, perceptions, and attitudes towards mathematics and its teaching and learning

- Teachers' beliefs and perceptions about the nature of mathematics
- Teachers' beliefs and perceptions about mathematics teaching and learning
- Challenging teachers' beliefs and perceptions
- Conceptual learning
- Contextual learning

## Unit 3: Exploring mathematical processes through a review of classroom-based research

- Review of classroom-based research studies conducted in the Western as well as the local context to identify best practices
- Analysis of the research studies to explore how to teach mathematical content to students, using a variety of teaching techniques and methods
- Implications for Student Teachers

## Unit 4: Planning for teaching

- Assessment for learning and assessment of learning
- Designing mathematical tasks and an assessment of mathematical content to facilitate students' learning
- Unit planning, with detailed lesson planning
- Classroom management (behaviour, time, and resources) and ways to handle students' responses
- Integration of ICTs

## Unit 1: The nature, nurture, and scope of mathematics

A philosophical basis defines the nature and scope of a discipline. In this unit, Student Teachers will examine the different philosophies of mathematics. They will identify possible connections and influences on mathematics teaching and learning. This unit will also cover development in the subject of mathematics over of time.

## Learning outcomes

After completing this unit, Student Teachers will be able to:

- explore different schools of thought, such as absolutist, fallibilist, constructivist, and social constructivist
- identify possible connections between and influences of perspectives on the nature of mathematics and its teaching and learning
- differentiate between the different approaches to teaching mathematics that develop instrumental and conceptual understanding
- relate the importance of mathematics in daily life
- explain the relationship of mathematics to other subjects
- critically analyse mathematics content and students' learning outcomes in light of the mathematics philosophy proposed in the National Curriculum for Mathematics (Grades I–VIII).

### Week 1: Introduction to the course

- Reviewing previous courses in light of mathematics concepts and processes learnt
- Assessing Student Teachers' understanding of previous courses in mathematics
- Sharing the course outline and outcomes

### Week 2: The nature and significance of mathematics

The nature of mathematics: absolutist, fallibilist, constructivist, and social constructivist view

### Week 3: Instrumental versus relational understanding

- Instrumental and relational understanding
- Exploring mathematics concepts, rules, and formulae to develop conceptual understanding

### Week 4: National Curriculum for Mathematics (2006) (Grades I–VIII)

- The use of mathematics learning in daily life
- Identifying the underlying philosophy of mathematics in curriculum standards and benchmarks
- Student learning outcomes defined in the National Curriculum
- Aligning the student learning outcomes with approved textbooks and other resources
- The relationship of mathematics to other subjects



## Unit 2: Teacher beliefs, perceptions, and attitudes towards mathematics and its teaching and learning

This unit will help Student Teachers recognize that teachers' beliefs influence their practices. It will give ample opportunity to challenge their own beliefs, perceptions, and attitudes toward mathematics and its teaching and learning.

### Learning outcomes

After completing this unit, Student Teachers will be able to:

- discuss how teachers' beliefs, perceptions, and attitudes influence their teaching practice
- list the common misconceptions about teaching and learning mathematics
- critically review their own beliefs and attitudes towards teaching and learning mathematics and discuss how to develop students' conceptual understanding
- develop teaching activities from their own context for teaching mathematical concepts
- use the developed teaching activities for the progression of mathematical concepts.

### Weeks 5 and 6: Teachers' beliefs, perceptions, and attitudes

- Defining beliefs, perceptions, and attitudes and discussing their effects on students' learning
- Reviewing research studies conducted in both the Western and the local context in order to identify mathematics teachers' beliefs about mathematics and its teaching and learning
- Identifying common misconceptions people generally have about (learning) mathematics

### Week 7: Challenging teachers' beliefs, perceptions, and attitudes

- Identifying their own beliefs and attitudes towards mathematics and its teaching and learning based on their learning experiences in school
- Challenging their own beliefs and attitudes towards mathematics and its teaching and learning

### Week 8: Conceptual learning

- What is conceptual learning?
- How does conceptual learning make mathematics meaningful?

### Week 9: Contextual learning

- Contextual learning
- How does contextual learning enhance understanding?
- Introducing different activities based on contextual learning

## Unit 3: Exploring mathematical processes through a review of classroom-based research

This unit will use recent articles explaining different mathematical processes related to elementary-level mathematics. After the overview of processes, Student Teachers will select best practices and present these to peers. Overall, this unit will help them link mathematics philosophy and teachers' beliefs with teaching practices to enhance students' conceptual understanding.

### Learning outcomes

After completing this unit, Student Teachers will be able to:

- review research articles relevant to teaching and learning mathematics
- discuss and elaborate on the basic mathematical processes identified in these articles
- find best practices in these research studies to be incorporated in the learning-teaching process in their own context.

### Weeks 10 and 11: Reflection on research papers

- Searching relevant research papers that discuss mathematics processes to teach for conceptual understanding
- Reviewing the identified research studies
- Discussing different teaching practices highlighted in the papers
- Writing key lessons learnt or a critical reflection on the reviewed research papers

### Week 12: Identification of best practices

- Discussing the benefits of practices indicated in research across the globe
- Discussing the usability of the identified practices in a Pakistani context in light of the National Curriculum for Mathematics
- Presenting some concrete examples on best practices for teaching mathematical concepts, rules, and formulae

## Unit 4: Unit planning

This unit focuses on using unit planning. It will provide an opportunity for Student Teachers to explore the importance of unit planning and also to develop unit plans for teaching mathematics concepts included in the National Curriculum for Mathematics. Different types of lesson designs will be emphasized in order to create variety in methods to explore mathematical concepts. The use of ICT, teaching strategies, and assessment techniques will also be discussed at length, so that Student Teachers will be at ease with the use of such technologies, techniques, and assessments.

## Learning outcomes

After completing this unit, Student Teachers will be able to:

- articulate the importance of unit planning
- develop unit plans by taking informed decisions about what to teach and how to teach and assess it
- develop lesson plans aligned with the unit aims and objectives, integrating appropriate ICTs and other teaching aids
- develop relevant assessment techniques to assess students' learning
- plan to manage resources and time effectively while implementing lesson plans.

## Week 13: Assessment techniques and their use in mathematics learning

- The difference between assessment and evaluation
- Understanding the purposes and tools of assessment
- Different types of assessment
  - Formative assessment
    - Portfolio
    - Project work
    - Mathematical investigation
  - Summative assessment
- Test and rubric construction
- Designing questions to promote thinking

## Week 14: The integration of ICTs

- Exploring mathematics concepts using ICT
- Identifying appropriate and relevant technologies that could facilitate mathematics learning

## Week 15: Classroom management

- Resource management – teaching resources, including ICT
- Time management
- Handling students' responses

## Week 16: Unit planning

- Key components of unit planning
- Different models of lesson planning
  - LES (Launch, Explore, and Summarize)
  - 5E (Engage, Explore, Explain, Elaborate, and Evaluate)
  - 4P (Preparation, Presentation, Practice, and Production)
  - MTA (Motivate, Teach, and Assess)
- Developing unit plans with integrated lesson plans to achieve the unit aims and objectives
- Micro-teaching: Delivering lessons to peers
- Reviewing the unit planning based on the feedback received

## Course assignments and assessment

Student Teachers will be assessed using both formative and summative assessments. Formative assessments will occur during coursework, such as using pencil-and-paper tests; quizzes and games – competition; instructor observation; peer observation; teacher or group projects; worksheets; simulations; portfolios; performance tasks; presentations, whether individual or group; and student self-assessment. The focus is on supporting Student Teacher to improve their learning process. With summative assessments, Student Teachers will be evaluated upon completion of the work and the focus will be on the written test.

## Assessment and grading

<b>1. Formative assessment</b>	<b>50%</b>
• Developing unit plans	20%
• Reviewing research articles (at least two)	20%
• Presentations (at least two)	10%
<b>2. Summative assessment</b>	<b>50%</b>

Note: Grades will be assigned as per the criteria of the university or institution.

## References

Textbooks, journal articles, and web resources are included in this section.

Amirali, M. (2010). Students' conceptions of the nature of mathematics and attitudes towards mathematics learning. *Journal of Research and Reflections in Education*, 4(1), 27–41.

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. International Bureau of Education: UNESCO. Retrieved from:

➤ [http://www.ibe.unesco.org/fileadmin/user\\_upload/.../EdPractices\\_19.pdf](http://www.ibe.unesco.org/fileadmin/user_upload/.../EdPractices_19.pdf)

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Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39–48). New York: Macmillan.

Ernest, P. (1994). *Constructing mathematical knowledge: Epistemology and mathematical education*. London: Taylor and Francis.

Kinach, B. M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary Mathematics Methods course: Toward a model of effective practice. *Teaching and Teacher Education*, 18, 51–71.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.

Thwaites, A. (2008). *100 ideas of teaching primary mathematics*. London: Continuum International Publishing.

Mathematical activities and lesson plans:

➤ <http://illuminations.nctm.org/>

Additional activities and lesson plans:

➤ <http://www.nctm.org/>

Research-based papers:

➤ [http://ecommons.aku.edu/pakistan\\_ied\\_pdck/](http://ecommons.aku.edu/pakistan_ied_pdck/)

➤



## SYLLABUS 2

### By

Dr Shafiq Ur Rehman, Dr Shahzada Qaiser, and Shumaila Hashim

### Year and semester

Year 4, Semester 7

### Credit value

3 credit hours

### Prerequisites

Methods of Teaching, Mathematics 1 and 2

## Course description

The course will focus initially on the processes of learning and teaching mathematics, and then consider the role of the teacher in enhancing learning. Student Teachers will review pedagogical approaches to mathematics and consider priorities for the future in learning and teaching mathematics. It will help them learn from their cultural context and link school mathematics with out-of-school mathematics. Theoretical approaches to learning mathematics will be examined and applied to better understand the key problems and challenges in mathematics education today, which include language issues, technology, contexts, feelings, beliefs, and attitudes. This course will provide Student Teachers with ICT knowledge, and they will look at ways in which ICT can successfully be applied to and integrated into the curriculum. It will help Student Teachers develop cognitive ability (reasoning, decision-making, and reflection) that may be useful to enhance the critical mathematical thinking of students in the context of practice. This course will also assist Student Teachers to develop low-cost resources individually and socially.

## Course outcomes

As a result of participation in this course, Student Teachers will be able to:

- acquire a deeper understanding of the ways in which learners learn and teachers teach mathematics, and of connections between learning and teaching mathematics
- develop skills of reading literature on mathematics education critically and of expressing arguments in mathematics education cogently
- challenge the beliefs, ideas, and perceptions about teaching and learning mathematics
- appraise theoretical approaches to learning and teaching mathematics and test these ideas in planning for teaching.

## Learning and teaching approaches

The following approaches will be used in the course:

- Activity-based teaching
- Inquiry method
- Discovery method
- Exploration method
- Demonstration method
- Lecture method
- Discussion with peers and Instructor
- Use of ICTs to facilitate learning and teaching

Also refer to the link available on the HEC website:

➤ [http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20\(Presentation\)/COMMON%20TEACHING%20METHODS.pdf](http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20(Presentation)/COMMON%20TEACHING%20METHODS.pdf)

The site provides a PowerPoint presentation on the above-mentioned methods of teaching, listing their strengths and weaknesses. This will help Student Teachers explore the advantages and disadvantages of teaching methods and take informed decisions on selecting appropriate teaching methods, considering the content and concept of teaching, the learning environment, and the resources available for teaching.

## Semester outline

### Unit 1: The nature of mathematics and mathematical thinking

- Different views and perspectives on mathematics and its teaching and learning (e.g. constructivist, humanistic, social constructivist, absolutist, and fallibilist)
- Teachers' conceptions of the nature of mathematics
- Beliefs that influence classroom practices
- Historical development in mathematics: Patterns and relationships

### Unit 2: The mathematical processes

- How children learn mathematics
- Understanding mathematical processes (critical thinking and abstract-concrete thinking)
- Methods of teaching mathematics
- Communication in the mathematics classroom
- Connections (establishing relationships among ideas and facts)
- Representation: Abstraction and symbolic representation

## Unit 3: Planning teaching and sequencing mathematical content and concepts

- Fundamentals of mathematical content sequencing
- Aligning mathematical content with other subjects
- Managing the mathematics classroom
- Bridging the gap between theory and practice
- Contextualizing content and tasks
- Managing resources
- Adopting or adapting research-informed strategies
- Developing unit plans, lesson plans, activities, and tasks: Enhancing critical-thinking skills
- Research-based decision-making
- Digital tools for learning mathematics

## Unit 1: The nature of mathematics and mathematical thinking (3 weeks)

### Description

In this unit, Student Teachers will explore the nature of mathematics and the beliefs, perspectives, and conceptions people have about it. These beliefs, perspectives, and conceptions are discussed in the context of the work of great philosophers and mathematicians. An understanding of different conceptions of mathematics is important to the development and successful implementation of mathematics education. It can be helpful for mathematics teachers to know how the field of mathematics has evolved and the interrelationships of different content areas. Student Teachers will be able to understand the power of mathematics, and the ways our society understands and reacts to the ever-widening influence of mathematics on our daily lives.

### Unit outcomes

This unit will enable Student Teachers to:

- explore different views and perspectives on mathematics and its teaching and learning, such as constructivist, humanistic, social constructivist, absolutist, and fallibilist
- identify possible connections between and the influences of the nature of mathematics upon mathematics teaching and learning
- acquire the necessary mathematical concepts and skills for everyday life and for continuous learning in mathematics and related disciplines
- develop the necessary process skills for the acquisition and application of mathematical concepts and skills
- develop mathematical thinking and problem-solving skills and apply these skills to formulate and solve problems
- develop the ability to reason logically, communicate mathematically, and learn cooperatively and independently.



## Week 1

Different views and perspectives on mathematics and its teaching and learning

- Absolutist and fallibilist
- Humanistic, constructivist, and social constructivist
- Instrumentalist and Platonic view of mathematics
- Historical development in mathematics: Patterns and relationships
  - How has mathematics evolved?
  - Is mathematics discovered or invented?

## Weeks 2 and 3

- Teachers' conceptions about the nature of mathematics
- Beliefs that influence classroom practice
- Challenging teachers' beliefs about mathematics
- Exploring mathematics concepts, rules, and formulae to learn for conceptual understanding

## Unit 2: Mathematical processes (7 weeks)

### Description

This unit will consider some of the processes that seem to apply to mathematical challenges, and ask whether what is involved in general problem-solving applies to mathematics. What is the mechanism by which teachers are supposed to base their teaching on research? And what actually happens in practice when teachers use research in their teaching?

This unit considers the problem and what might be done to create links between settings and linguistic and attitudinal factors that underlie learning difficulties in mathematics, together with ways in which the teacher might improve matters and norms. Furthermore, an array of the most common student misconceptions will be examined. Ways to facilitate students' learning of mathematics for conceptual understanding will be discussed.

## Unit outcomes

This unit will enable Student Teachers to:

- understand different mathematical processes
- identify and practice different problem-solving and logical-reasoning techniques
- draw upon their knowledge of the concepts and skills they have learned in the earlier courses to be successful problem solvers
- discuss ways to handle communication in the mathematics classroom
- make effective use of a variety of mathematical tools (including ICT tools) in the learning and application of mathematics
- recognize and use connections between mathematical ideas and between mathematics and other disciplines
- develop positive attitudes towards mathematics
- produce imaginative and creative work arising from mathematical ideas
- build up mathematical terminology.

## Weeks 4 and 5

How children learn mathematics

- Insights Student Teachers must have in order to understand basic mathematical concepts
- Understanding mathematical processes
- Concept formation
- Critical thinking
- 'Hands-on' and 'minds-on' activities
- Abstract and concrete experiences
- The cognitive demands of mathematical tasks

## Week 6

Methods of teaching mathematics

- Deductive method
- Inductive method
- Problem-solving
- Learning mathematics with manipulatives and visual aids

## Week 7

Methods of teaching mathematics (cont.)

- Project method
- Heuristic method
- Peer-teaching
- Demonstration

## Week 8

Exploring mathematical concepts using ICT in mathematics: Digital tools for learning mathematics

- Incorporating ICT in mathematics teaching
- Encouraging mathematics teachers to use ICT
- Locating resources for using ICT

## Week 9

- Communication in the mathematics classroom
- Teaching core vocabulary in advance
- Connections (establishing relationships among ideas and facts)
- Representation: Abstraction and symbolic representation

## Unit 3: Planning teaching and sequencing mathematical content (6 weeks)

### Description

This unit is related to the actual planning for the dissemination of pedagogical content knowledge and skills in mathematics. It helps Student Teachers develop, plan, and implement the required tasks and activities. It will also guide them in managing, using, and developing resources. This unit will try to contextualize tasks and activities according to the local environment. In the end, it takes effort to fill the gap between theory and practice in mathematics education.

### Unit outcomes

This unit will enable Student Teachers to:

- learn the basic rules for mathematical sequencing in light of the National Curriculum for Mathematics
- develop unit plans by taking informed decisions about what to teach and how to teach and assess it
- develop lesson plans aligned with the unit aims and objectives, integrating appropriate ICTs and other teaching aids
- develop relevant assessment items to assess students' learning
- plan to manage resources and time effectively while implementing lesson plans
- develop low-cost resources for learning mathematics for conceptual understanding
- identify the strengths available in the local context and utilize them in planning for teaching
- effectively plan, develop, and implement tasks and activities in their classrooms
- set up a mathematics classroom that can facilitate students' learning
- plan tasks and activities using ICT tools in teaching mathematics
- review the research on teaching mathematics.

## Week 10

- Building an understanding of the National Curriculum of Mathematics
- Fundamentals of mathematical content sequencing
  - Relating different mathematical contents and concepts
  - Prerequisites and expertise required for different mathematical content areas

## Week 11

Bridging the gap between theory and practice

- Identifying current teaching practices using research on mathematics education
- What does research say about teaching approaches?

## Week 12

- The difference between assessment and evaluation
- Understanding the purposes and tools of assessment
- Different types of assessment
  - Formative assessment
    - Portfolio
    - Project work
    - Mathematical investigation
    - Summative assessment
- Test and rubric construction
- Designing questions to promote thinking

## Week 13

Contextualizing the content and tasks

- Identifying the student's local context
- Making language comprehensible in the local context
- Designing the learning environment in the local context
  - Learner-centred environment
  - Knowledge-centred environment
  - Assessment-centred environment
  - Community-centred environment

## Week 14

- Resource management
  - Mathematics laboratory
  - ICT
- Time management
- Handling students' responses

## Weeks 15 and 16

- Key components of unit planning
- Different models of lesson planning
  - LES (Launch, Explore, and Summarize)
  - 5E (Engage, Explore, Explain, Elaborate, and Evaluate)
  - 4P (Preparation, Presentation, Practice, and Production)
  - MTA (Motivate, Teach, and Assess)
- Developing unit plans with integrated lesson plans to achieve the unit aims and objectives
- Micro- or peer-teaching
- Reviewing the unit planning based on the received feedback

## Course assignments

### Group assignment

Choose any mathematician of your choice and prepare a term paper detailing his/her:

- contributions to the field of mathematics
- beliefs, views, conceptions, and ideas about the nature of mathematics and teaching mathematics.

### Individual assignments

1. Select a concept from the National Curriculum of Elementary Mathematics and prepare a unit plan, including detailed lesson plans with activities, resources, and reading material.
2. Present the unit plan.
3. Review research-based articles and articulate lessons learnt for teaching and learning mathematics.
4. Summative test

## Grading policy

A variety of assessments will be used, including midterm and final examinations.

Ongoing assignments	60%
Midterm and final term test	40%

## References

Textbooks, journal articles, and web resources are included in this section.

Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A survey of secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research*, 32(2), 45–61. Retrieved from:

➤ <http://pu.edu.pk/images/journal/ier/PDF-FILES/3-Munira%20Almirani.pdf>

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Confery, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *Educational Studies in Mathematics*, 26, 135–164.

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Lerman, S. (1990). Alternative perspectives of the nature of mathematics and their influence on the teaching of mathematics. *British Educational Research Journal*, 16(1), 53–61.

Sfard, A., & Linchevski, L. (1994). Between arithmetic and algebra: In the search of a missing link. The case of equations and inequalities. *Rendiconti del Seminario Matematico*, 52(3), 279–307.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.

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# SYLLABUS 3



## By

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and Shoaib Mohsin Ali

## Year and semester

Year 4, Semester 7

## Credit value

3 credits

## Prerequisites

Mathematics 1 and 2 and Methods of Teaching

## Course description

Teaching mathematics as a science of logic is one of the demands of the 21st century. For this reason, teachers of mathematics are deemed more important than ever. We have teachers who are good practitioners of mathematics, but we need teachers who are competent in the science of teaching mathematics.

Teaching mathematics is not an end in itself; it is rather a means to a greater end. This course is an attempt to bring to light those aspects of mathematics pedagogy that will change the beliefs, conceptions, and attitudes of Student Teachers about the teaching of mathematics. Topics and methodologies have been included to help Student Teachers gain those competencies they need to develop better strategies for the teaching of mathematics.

This course provides opportunities for teachers in elementary grades to bridge the gap between theory and practice, as well as mathematical content knowledge and pedagogy. It helps them develop pedagogical content knowledge by challenging their beliefs about the process of learning mathematics. It also complements their competencies by incorporating research-based best practices.

## Course outcomes

By the end of this course, Student Teachers will be able to:

- demonstrate practical mathematical skills
- bridge the gap between content knowledge and pedagogy
- discuss misconceptions and beliefs in the teaching of mathematics
- make use of research-based best practices in their work.

## Learning and teaching approaches

The following approaches will be used in the course:

- Activity-based teaching
- Inquiry method
- Discovery method
- Exploration method
- Demonstration method
- Lecture method
- Discussion with peers and Instructor
- Use of ICTs to facilitate learning and teaching

Also refer to the link available on the HEC website:

➤ [http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20\(Presentation\)/COMMON%20TEACHING%20METHODS.pdf](http://www.hec.gov.pk/InsideHEC/Divisions/LearningInnovation/Documents/Learning%20Portal/NAHE%20(Presentation)/COMMON%20TEACHING%20METHODS.pdf)

The site provides a PowerPoint presentation on the above-mentioned methods of teaching, listing their strengths and weaknesses. This will help Student Teachers explore the advantages and disadvantages of teaching methods and take informed decisions on selecting appropriate teaching methods, considering the content and concept of teaching, the learning environment, and the resources available for teaching. This course will include lectures and handouts by the course Instructor coupled with in-class activities, reading, and discussions, as well as home assignments, project work, and reviewing the suggested research-based studies.

## Semester outline

### Unit 1: The nature of mathematics education

#### Description

The unit focuses on various mathematicians' perspectives on the nature of mathematics. It explores prevalent beliefs and consequently misconceptions about the teaching of mathematics. This will help Student Teachers develop their understanding of learners' difficulties in mathematics. They will learn how to communicate mathematics to learners in order to enhance and enrich the development of students' understanding of mathematics in primary school.

#### Learning outcomes

Student Teachers will be able to:

- assimilate different perspectives on the nature of mathematics
- review various historical developments in mathematics
- identify and understand various beliefs about mathematics education
- understand the implications of teachers' beliefs for their practices.



## Week 1

Different perspectives on the nature of mathematics

## Week 2

Historical review of the development of mathematics and mathematics education

Conceptions and beliefs about learning mathematics

## Week 3

Teachers' beliefs and the influence of these beliefs on their practices

## Unit 2: Processes of mathematics education

### Description

This unit focuses on mathematical processes and classroom practices that support and develop children's mathematical thinking. Student Teachers will also develop frameworks for assessing children's learning. Tactics and various models for solving problems will be introduced while exploring elementary arithmetic, algebra, and measurement concepts. Examples include understanding the place value concept, the structure of the base ten in the number system, and the meaning of the four operations. This investigation will help them understand the connection between arithmetical and algebraic thinking in the elementary grades. The key goal for each mathematical idea is to understand how tasks and classroom practices can be structured to encourage student thinking and how the learning environment can support the development of students' mathematical ideas and identities. In particular, Student Teachers will learn how to use a set of instructional activities in a mathematics classroom to help children develop their skills, such as computational fluency, and articulate their mathematical thinking.

### Learning outcomes

Student Teachers will be able to:

- study mathematics content and mathematics pedagogy for conceptual understanding
- understand that what students learn is fundamentally connected to how they learn it
- understand and use various methods and processes appropriate for teaching students of different abilities
- understand that the vision of mathematics teaching is to teach through problem-solving, which is not an isolated concept but a process
- learn how and when to use different strategies, models, and tactics in different situations.

## Weeks 4 and 5

Teaching methods and strategies

In this unit the following methods have been discussed:

- Dogmatic method
- Inductive-deductive method
- Analytic-synthetic method
- Laboratory method

The following skills will be developed through engaging Student Teachers while exploring mathematical concepts, problem-solving, and pattern-seeking tasks:

- Specializing
- Generalizing
- Conjecturing
- Convincing

## Week 6

Techniques and devices

Tactics (this is a part of strategies – understanding the tactics of investigating mathematical concepts)

## Unit 3: Tasks and resource development

### Description

Teachers are the most important resource for developing students' mathematical abilities. They are expected to raise comfort levels and give students confidence in their capacity to learn mathematics. This unit explores innovative and interrelated factors that can boost skills for attempting mathematical problems and enhance their ability to work independently and collaboratively. It will open up new ways of thought through a continuation of brainstorming processes. Student Teachers will learn techniques to develop resources with what they have available and create tasks that enhance conceptual understanding of mathematics among students in an interactive way. They will practice developing tasks to clarify their students' understanding and broader interpretation of mathematical ideas. Assessment criteria for these tasks will also be taught.

### Learning outcomes

Student Teachers will be able to:

- provide students with broader or new angles to view the mathematics learning context
- develop low-cost learning resources
- use the available resources effectively
- build understanding among students
- generate strong, thought-provoking ideas and use them for learning.

## Week 7

Identification and selection of tasks and resources for a specific topic, e.g. teaching algebra

## Week 8

Sequencing of the tasks and resources according to the classroom situation and the needs of Student Teachers on the one hand and the methods and techniques on the other

## Week 9

Development of required low-cost or no-cost resources

## Week 10

Development of a mathematics laboratory

## Unit 4: Planning for teaching

### Description

This unit gives the rationale of the learning objectives and their classification. In-depth understanding of these taxonomies will help Student Teachers to define their targets in a more practical, achievable, and measurable way. Both of the taxonomies have their own intrinsic values in the teaching-learning process. Student Teachers are shown how to plan for teaching mathematics. The planning ranges from macro- to micro-level. It includes planning the course for the entire semester, unit planning, and lesson planning, along with the observation protocol that assures the execution of the plan and the achievement of the learning outcomes. Student Teachers are then given teaching practice before going to teach in schools.

### Learning outcomes

Student Teachers will be able to:

- understand and appreciate the classification of educational objectives
- develop and implement a comprehensive plan for the semester, course, and session (along with the observation protocols)
- discuss and provide feedback on practical experiences of teaching and learning mathematics in a micro-teaching situation

## Week 11

Taxonomies of educational objectives (Bloom's taxonomy and SOLO taxonomy)

Assessment (consult the course on Assessment for Semester 4)

Developing effective assessment items

## Week 12

Planning for teaching mathematics

Planner (semester planning and term syllabus)

Unit planning

Lesson of the day

Observation protocol (rubrics, etc.)

## Week 13

Demonstration by the Student Teachers (micro-teaching and dramatizing the whole process in the classroom)

Feedback and discussion

## Week 14

Continuation of demonstration by the Student Teachers (micro-teaching and dramatizing the whole process in the classroom)

Feedback and discussion

## Unit 5: Classroom management

### Description

This unit provides higher-order learning to the Student Teachers. It begins with meta-teaching. Concrete (e.g. logistics) as well as abstract (e.g. ethical standards, practical difficulties, and research findings) aspects of the teaching of mathematics are discussed. The unit ends with an overview of the judicious use of ICT, and prepares Student Teachers to avoid the problems and overcome the practical difficulties of ICT, such as power failures.

### Learning outcomes

Student Teachers will be able to:

- review the research studies done in the relevant context
- assimilate a code of conduct for the course Instructor as well as the Student Teachers
- use the knowledge and skills of classroom management for the mathematics classroom
- understand the implications of using ICT in an indigenous setting.

## Week 15

Research-based best practices

Code of conduct in the classroom

Ethics for the student

Ethics for the teacher

Classroom physical environment (logistics)

Team-teaching (difficulties?)

Exposure to successful senior teachers of mathematics (team-building?)

Case studies

Action research outcomes

## Week 16

ICT in teaching mathematics (Cabri software, graphical calculators, interactive websites, and MS Excel)

What is ICT?

Promoting a learning environment for using ICT

Implications (advantages and disadvantages) of using ICT

How to deal with ICT problems (power failure or breakage)

## Course assignments and assessment

Student Teachers will be given individual and collective assignments; these will contribute towards their final grade. They will be assessed using both formative and summative assessments. With formative assessments, they will be assessed during the coursework and the focus will be on improving their learning process. With summative assessments, they will be evaluated upon completion of the work and the focus will be on the written test.

### Assessment and grading

<b>1. Formative assessment</b>	<b>50%</b>
• Developing unit plans	20%
• Reviewing research articles	20%
• Presentations	10%
<b>2. Summative assessment</b>	<b>40%</b>
<b>3. Homework portfolio</b>	<b>10%</b>

Note: Grades will be assigned as per the criteria of the university or institution.

## References

Textbooks, journal articles, and web resources are included in this section.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83–104). Westport, CT: Ablex.

Ball, D. L., & Cohen, D. K. (Eds.). (1999). *Developing practice, developing practitioners: Towards a practice-based theory of professional education*. San Francisco: Jossey-Bass Publishers.

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Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), D. A. (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39–48). New York: Macmillan.

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Ouviney, R. J. (1989). *Learning to teach mathematics*. Melbourne: Merrill Publishing Co.

Sidhu, K. S. (1989). *The teaching of mathematics*. New Delhi: Sterling Publishers.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Touchstone.

3

# Integrated teaching notes



During the curriculum development process, faculty were encouraged to keep notes that would be useful to them and others who may teach the course in the future. These were submitted along with the course syllabus. Teaching notes include ways to introduce the course, ideas for teaching units and sessions, sample lessons plans, and suggestions for reading and resource material. These have been integrated into a single section of this document to create a rich and varied collection of ideas easily accessible to others. The section is organized by theme.

## Focus of the Mathematics III course

The two key aspects on which Instructors need to focus throughout the course are:

### Conceptual understanding

The main focus of the teacher learning sessions is to enable Student Teachers to recognize the importance of conceptual understanding in learning mathematics. This does not mean that there is no place for procedural understanding. It is important to acknowledge that learners need ample opportunities to explore concepts rather than perform drills and practice the rules, formulae, and procedures given by the teacher. Classroom-based research shows that when learners explore mathematical rules and formulae by themselves, they develop a positive attitude toward mathematics and see connections between different mathematical concepts. The following task list includes some of the mathematical concepts of which a conceptual understanding is required.

1. Why do we add the numerator to the numerator but not the denominator to the denominator when two fractions with a common denominator are added?
2. Why do we multiply the numerator by the numerator and the denominator by the denominator when two fractions are multiplied?
3. In fraction division, why we do first turn the dividend upside down and then multiply it?
4. Why does 0.3 times 0.2 equal 0.06 and not 0.6?
5. Why is a minus times a minus always a plus?
6. Why is an absolute value always a positive number?
7. Why is the product of means equal to the product of extremes?
8. Why does a number divided by zero lead to infinity?
9. Why is the value of pi  $22/7$ ?
10. Why is  $\sqrt{\quad}$  referred to as  $\frac{1}{2}$ ?
11. Why does  $\sqrt{2} \times \sqrt{2}$  equal 2?
12. Why does  $x^0$  equal 1?
13. Why is the rule to calculate the number of a possible subset  $2^n$ ?
14. What is the logic behind the square root by division method?



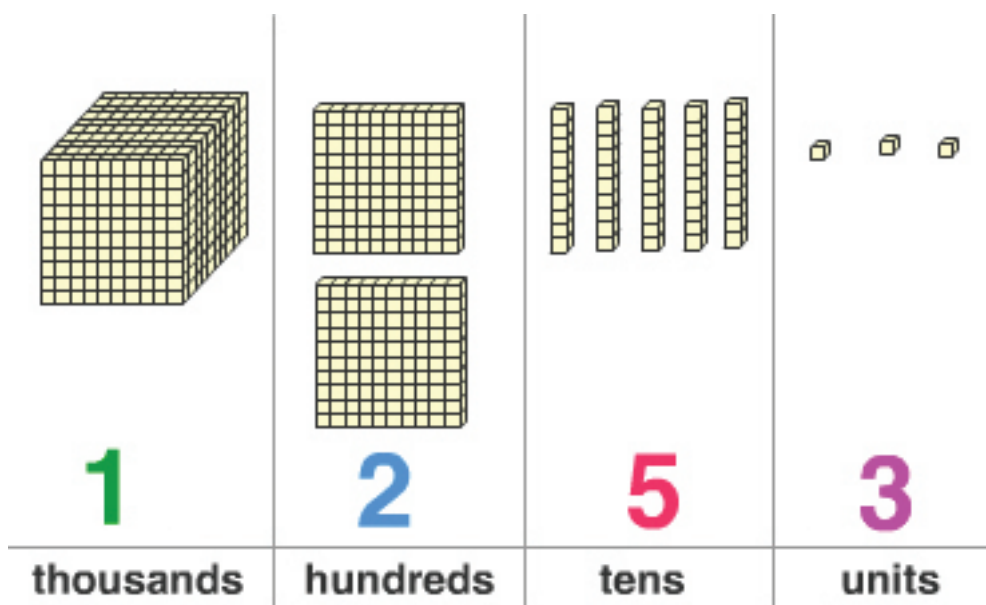
## Practical work

To develop mathematical concepts, Student Teachers need to interact with concrete and semi-concrete materials before they interact with symbolic presentation. This will help them realize that hand-on and minds-on tasks are fundamental to the development of mathematics understanding at all levels. Learners at all levels can benefit from practical work if this is appropriately selected, well planned, and relevant to the concepts. The teacher's role is significant in selecting appropriate resources and giving clear instructions to the learners. If a teacher does not select appropriate resources or activities or give clear instructions, this can hinder the learning process instead of benefiting learners. The use of mathematical software and online activities can also play an important role in developing conceptual understanding in mathematics. A suggested list of resources that can be used in this course is given below.

## Concrete and semi-concrete materials

Place value blocks	Physical balance
Number cards	Pair of scissors
Multi-link cubes or different-coloured counters	Masking tape
Fraction blocks	Weight scale
Tangram	Measuring tape or scale
Abacus model	Measuring cylinders or beakers
2D shapes or 3D objects	Geoboard
Dotted papers	Board geometry box
Square papers	Graph board
Paper and graph paper	Rope or thread

An example to teach the place value concept using place value blocks is shown in the following grid.



## Basic principles for school mathematics

The following are the key principles for school teaching and learning of mathematics:

- Each and every child can learn mathematics.
- Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Mathematical thinking is supported by using a variety of teaching modes and resources.
- Effective mathematics teaching requires an understanding of what students know and need to learn and then challenging and supporting them to learn it well.
- Assessment should support students' learning and provide useful information to both teachers and students.
- Students can learn mathematics more effectively with the appropriate use of technology. When technological tools are available, students can focus on decision-making, reflection, reasoning, and problem-solving.

## Unit on the nature, nurture, and scope of mathematics

This unit is present in all three of the above syllabi. This section contains integrated teaching notes where the ideas shared by the faculty who participated in planning this course has been incorporated. Teaching notes include ways to introduce the course, ideas for teaching units and sessions, lesson plans, and suggestions for reading and resource material. The teaching notes have been integrated around broad themes addressed in the course. Faculty who are teaching the course for the first time or who are interested in the process of curriculum design may find it useful to see how the concepts are being developed.

A philosophical basis defines the nature and scope of a discipline. In this unit, Student Teachers will examine the different philosophies of mathematics. They will identify possible connections and influences on mathematics teaching and learning. This unit will also cover development in the subject of mathematics over of time.

<sup>3</sup> Adapted from Mason, J. (1993). *Thinking mathematically*, and the National Council of Teachers of Mathematics website: <http://www.nctm.org/standards/content.aspx?id=26802>

## Learning outcomes

After completing this unit, Student Teachers will be able to:

- explore different schools of thought, such as absolutist, fallibilist, constructivist, and social constructivist
- identify possible connections between and influences of perspectives on the nature of mathematics and its teaching and learning
- differentiate between the different approaches to teaching mathematics that develop instrumental and conceptual understanding
- relate the importance of mathematics in daily life
- explain the relationship of mathematics to other subjects
- critically analyse mathematics content and student learning outcomes in light of the mathematics philosophy proposed in the National Curriculum for Mathematics (Grades I–VIII).

## Week 1: Introduction to the course



### Session 1: Sharing course outline (1 hour)

#### **Introduction (10 minutes)**

Ask Student Teachers to introduce themselves and briefly answer the question, ‘How did you feel about mathematics when you were studying in school?’ Take note of their feelings – you could refer to them later while discussing factors that influence students’ attitude towards mathematics.

#### **Ice-breaking (10 minutes)**

For a few seconds, show the Student Teachers a chart with a large number written on it, such as 371115192327, and ask them to write this number down. They will probably come up with different numbers. Discuss the reason why they failed and how they can be sure which number is correct. Note that at times even we adults find it difficult to memorize a number. One of the reasons is that we did not get a chance to build connections and identify a pattern, which would have helped us remember the number without much difficulty.

Next, take the Student Teachers through a process by asking them to identify the relationship between the numbers 3 and 7 (starting from the first number on the chart). Show them how the large number is generated by adding 4 to the previous number:  $3(+4)7(+4)11(+4)15(+4)19(+4)23(+4)27$ .

Ask them to share their key learning from this activity. Expected responses are as follows:

- When students understand the process, they enjoy learning mathematics.
- When they understand how a mathematical rule or formula is derived, they find mathematics meaningful.
- The teacher can play a key role in facilitating students’ learning process.

Sum up the discussion, noting that when students memorize facts and procedures without understanding, they often find mathematics tedious and boring. When they are engaged in hands-on and minds-on tasks, exploring, investigating, and identifying relationships, they develop conceptual understanding and mathematics starts making sense to them.

### **Activity: Course outcomes and outline (40 minutes)**

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Ask Student Teachers to share their expectations from this course. Then distribute the course description, learning outcomes, and session outline and ask Student Teachers to read and share the key ideas with the class. Write their responses on the board. Finally, discuss the learning outcomes and the main ideas in teaching and learning mathematics with Student Teachers. Wherever possible, refer to their expectations shared earlier. Encourage them to ask any questions for clarification.

### **Session 2: Pre-test (1 hour)**

The key purpose to a pre-test is to know whether the course participants can articulate a conceptual understanding of the mathematical rules and formulae they learnt in the previous course. If the Student Teachers are not able to respond, encourage them to form groups and explore the concept using hands-on and exploration tasks.

#### **Activity 1 (30 minutes)**

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Conduct the pre-test (see Reading and Resources – Pre-Test.)

#### **Activity 2 (30 minutes)**

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Engage Student Teachers in exploring the pre-test item on discovering the area of a circle to experience generalising a formula based on hands-on and minds-on activity. (Refer to the detailed lesson plan for discovering the area of a circle in the Reading and Resources section).

In the next week, mark the answer scripts and use the results to provide further support to Student Teachers. Share the results with them individually to enable them to identify their errors and correct them.

### **Session 3: Learning theories (1 hour)**

#### **Activity 1 (10 minutes)**

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Ask Student Teachers to recall how they learnt about the area of a circle in school and in the previous session in this course.

#### **Activity 2 (50 minutes)**

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Ask Student Teachers to read the description of behaviourism, cognitivism, and constructivism, and share the key aspects of each learning theory. After the discussion, sum up each theory by discussing its key aspects. Then ask Student Teachers to revisit their response to the previous task on discovering the area of a circle and see which learning theory was mostly promoted by their mathematics teachers in school and by their Instructor in the previous session.

Discuss the importance of engaging students in constructing mathematical knowledge. Follow this by a discussion of the underlying learning theory that the Teaching of Mathematics course wants Student Teachers to promote while teaching and learning mathematics.

NOTE: You need to develop a handout on learning theories for the class discussion. The following websites can assist you in preparing handouts for the class. Please include proper attribution to these sources.

Learning theories and models:

➤ <http://www.learning-theories.com/>

Learning theories overview:

➤ <http://www.cs.uci.ac.cy/~nicolast/courses/cs654/lectures/LearningTheories.pdf>

Learning theories and decision-making:

➤ <http://www.iacad.org/istj/36/2/editorial.pdf>

## Home reading

Give Student Teachers the following research article: Amirali, M. (2010). Students' conceptions of the nature of mathematics and attitudes towards mathematics learning. *Journal of Research and Reflections in Education*, 4(1), 27–41. (See Readings and Resources.)

## Week 2: The nature of mathematics



### Session 1: The nature and significance of mathematics (1 hour)

#### Activity (15 minutes): What is mathematics?

Ask Student Teachers to individually come up with a definition of mathematics, share their responses with their group, and then create a group definition. Share a handout on the definition of mathematics (see Readings and Resources) with each group. Ask them to find similarities between their definition and the definition shared by different mathematicians. Conclude this activity by restating what mathematics is.

#### Activity (45 minutes): Is mathematics discovered or invented?

Ask Student Teachers to write individual responses to a survey questionnaire (excerpts from the survey questionnaire developed by Dr Munira Amirali for her Ph.D. research are given in the Readings and Resources section) and then discuss these with the group. Share with them that two extreme views exist on the nature of mathematics, namely the absolutist view and the fallibilist view. Let them read the following article: Ernest, P. (n.d.). *Is mathematics discovered or invented?* Retrieved from:  
➤ <http://people.exeter.ac.uk/PErnest/pome12/article2.htm>

#### Home task

Ask Student Teachers to read the following article: Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A survey of secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research*, 32(2), 45–61. Retrieved from:  
➤ <http://pu.edu.pk/images/journal/ier/PDF-FILES/3-Munira%20Almirani.pdf>

## Session 2: The nature and significance of mathematics (cont.) (1 hour)

### Activity (30 minutes)

Conduct a discussion on the article ‘Teachers’ Knowledge about the Nature of Mathematics: A Survey of Secondary School Teachers in Karachi, Pakistan’. Ask Student Teachers to share different philosophical views on the nature of mathematics, as discussed in the article, first in a small group and then with the whole class. Then ask them to discuss the findings and why it is important to explore teachers’ conception of the nature of mathematics.

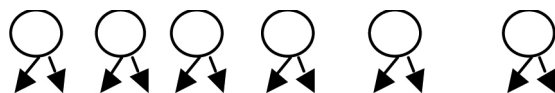
### Activity (30 minutes): Multiple ways to solve mathematical tasks

Keeping in mind the misconceptions identified in the course, involve Student Teachers in solving the following task to help them experience knowledge construction and multiple ways of solving a task.

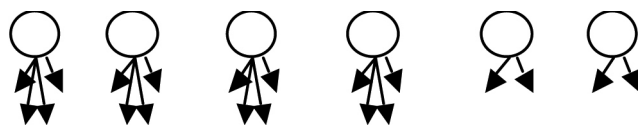
Azam keeps hens and rabbits in a cage. He goes to the cage to count them, and counts 6 heads and 20 legs. How many hens and rabbits were in the cage?

Encourage Student Teachers to explore different ways of solving the task. For instance, if this task is given to grade 3 students, how will they solve it? Ask them to share the process with the class. Conclude that this task can be solved through pattern-seeking, drawing, framing simultaneous equations, and so forth. An example is presented of how this task can be solved using drawing.

1. First step: 6 heads, 2 legs each



2. Second step: Remaining 8 legs



3. This shows there are 4 rabbits and 2 hens in the cage.

Discuss the importance of process in mathematics learning, rather than focusing only on getting the right answer.

## Session 3: Knowledge, skills, and attitude (1 hour)

### **Activity 1 (50 minutes): List the knowledge, skills, and attitudes required**

Ask Student Teachers to list what knowledge, skills, and attitudes mathematics teachers require to help students acquire conceptual understanding and build a positive attitude toward mathematics learning. After they have recorded their response, ask them to share in their respective groups followed by whole-class sharing. Highlight the key aspects in order to help them realize that they need to work hard to enhance their knowledge, skills, and attitudes to become effective teachers. Share the general finding from the pre-test and emphasize the importance of conceptual understanding and problem-solving in teaching and learning mathematics.

NOTE: Read the article on 'Subject and Pedagogical Content Knowledge for Teaching Mathematics', written by McNamara, O., Jaworski, B., Rowland, T., Hodgen, J., and Prestage, S. This chapter describes and synthesizes philosophical and empirical research on the different kinds of knowledge on which a teacher can draw in teaching mathematics. The theoretical framework draws on the seminal work of Lee Shulman, and the authors emphasize the kinds of knowledge most strongly associated with subject matter per se (in this case mathematics) and how learners might engage with and come to know mathematics. The authors provide accounts of studies in the United Kingdom into the character and extent of such knowledge, and how it relates to teaching in the classroom. They also include a discussion on the role of teachers' attitudes to and beliefs about mathematics. This chapter is available on:

➤ <http://www.maths-ed.org.uk/mathsteachdev/pdf/mdevc2.pdf>

### **Activity (10 minutes) Knowledge, skills, and attitudes**

Ask Student Teachers to write down the required knowledge, skills, and attitudes in light of the discussion and identify key aspects they want to develop to become effective mathematics teachers.

## Week 3: Effective pedagogies for mathematics teaching



### Session 1: National Curriculum for Mathematics (2006) (1 hour)

#### **Activity (60 minutes): National Curriculum for Mathematics (2006) suggested teaching strategies**

Let Student Teachers read the National Curriculum for Mathematics (2006) teaching strategies (see Readings and Resources) available on the Ministry of Education website.

Generate a detailed discussion to understand the role of the teacher in implementing the National Curriculum for Mathematics and the suggested approaches to engaging students in learning mathematics. Refer to the instructional strategies or methods, such as student-led activities, teacher-guided activities, investigations, discussions, worksheets, web-based activities, project-based activities, demonstrations, student research, data collection activities, and computer-aided activities.

## Home reading

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Ask Student Teachers to call out numbers 1 to 10 and assign each number a section to read from Anthony, G., & Walshaw, M. (2009). *Effective pedagogies for mathematics teaching*. Retrieved from [http://www.ibe.unesco.org/fileadmin/user\\_upload/Publications/Educational\\_Practices/EdPractices\\_19.pdf](http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf) for discussion in the following weeks. The document is free to use, and is reproduced in the Readings and Resources section.

Group 1: An ethic of care, pp. 7–8

Group 2: Arranging for learning, pp. 9–10

Group 3: Building on students' thinking, pp. 11–12

Group 4: Worthwhile mathematical tasks, pp. 13–14

Group 5: Making connections, pp. 15–16

Group 6: Assessment for learning, pp. 17–18

Group 7: Mathematical communication, pp. 19–20

Group 8: Mathematical language, pp. 21–22

Group 9: Tools and representations, pp. 23–24

Group 10: Teacher knowledge, pp. 25–26

## Session 2: Reading and discussion on effective pedagogies (1 hour)

Ask Student Teachers to first share the key points in small groups and then prepare a presentation for the class, bearing in mind that their colleagues do not have the text that their group has read.

## Session 3: Development of mathematical knowledge (1 hour)

### Activity 1 (30 minutes)

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Ask Student Teachers to discuss the following questions in small groups:

- How is mathematical knowledge developing?
- In your view, how has the concept of 'counting' developed?
- What was the need to use scientific notation, integers, negative numbers, and the like?
- How is mathematical knowledge facilitating modern technology and vice versa?

### Activity 2 (30 minutes): Whole-class discussion

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Engage Student Teachers in discussing their ideas at length. At the end of the session, summarize the discussion with the following key points:

- Mathematical knowledge emerged to facilitate real-life experiences.
- Mathematics formulae and rules were explored by people like us, so everyone can explore and make sense of mathematical knowledge.
- Mathematical knowledge is an ever-growing body of knowledge and is not static.
- Modern technology uses mathematical knowledge to progress.



# Teaching notes: Unit on mathematics processes

## Unit description

This unit focuses on mathematical processes and classroom practices that can support and develop children's mathematical thinking. Student Teachers will also develop frameworks for assessing children's learning. Tactics and various models for solving problems that involve specializing, generalizing, conjecturing, and convincing will be discussed. The unit will also explore the underlying rationale for rules and formulae in elementary grade arithmetic, algebra, and measurement, for instance understanding the place value concept, the structure of the base ten in the number system, and the meaning of the four operations. This investigation will help Student Teachers understand the connection between mathematical content included in the elementary grades. In particular, Student Teachers will learn how to use a set of instructional activities in a mathematics classroom to help children acquire the ability to think mathematically and to use mathematical thinking to solve problems.

## Learning outcomes

Student Teachers will be able to:

- recognize that the goal of mathematics teaching is to enable children to think mathematically
- recognize that problem-solving and mathematical thinking are not isolated concepts but rather are processes
- apply and adapt a variety of appropriate strategies to solve problems
- recognize reasoning and proof as fundamental aspects of mathematics learning

## Weekly outline, Unit 2: Processes of mathematics education

### Weeks 1 and 2

Teaching methods and strategies:

- Dogmatic method
- Inductive-deductive method
- Analytic-synthetic method
- Laboratory method

The following skills will be developed through engaging Student Teachers while exploring mathematical concepts and problem-solving and pattern-seeking tasks:

- Specializing
- Generalizing
- Conjecturing
- Convincing

## Week 3

Techniques and devices

Tactics (this is a part of strategies – understanding tactics for investigating mathematical concepts)



## Week 1: Methods of teaching mathematics

### Sessions 1 and 2: The importance of teaching methods (2 hours)

#### **Brainstorming (10 minutes)**

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Ask Student Teachers to recall some of the teaching strategies their mathematics teachers used while teaching. Encourage them to refer to the learning theories and teaching approaches studied in the earlier units to articulate their response.

#### **Activity (60 minutes): Discovering the area of a circle**

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Engage Student Teachers in discovering the area of a circle. Please refer to the detailed lesson plan in the Readings and Resources section. The lesson plan can also be accessed online at <http://illuminations.nctm.org/LessonDetail.aspx?ID=L577> (Used with permission.)

#### **Discussion (20 minutes)**

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Engage Student Teachers in discussing how the discovery of the area of a circle is different from what they studied in school. Also discuss the key aspects teachers need to consider while planning and teaching the lesson. Sum up by explaining the following key aspects of teachers' role in selecting an appropriate teaching method:

- Be aware of the advantages and disadvantages of different teaching methods and strategies while planning a lesson.
- Keep in mind the children's interest, their prior knowledge, and available resources in planning new knowledge, using appropriate teaching methods.
- Use a variety of instructional methods and techniques for helping learners construct mathematical knowledge.
- Provide students with an opportunity to learn.
- Listen carefully to students' ideas.
- Ask students to clarify and justify their ideas orally and in writing.
- Monitor students' participation in discussions and decide when and how to encourage each student to participate.

#### **Reading and discussion (30 minutes)**

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Divide Student Teachers into four groups, and give each group a handout (see Readings and Resources) related to the following topics: dogmatic method, inductive-deductive method, analytic-synthetic method, and laboratory method. Let them read and discuss it in their respective groups. Ask the groups to note the main points discussed and share them with the whole class. Summarize by highlighting the key aspects for each method discussed in the groups.

## Session 3: Exploring mathematical concepts through pattern-seeking (1 hour)

### **Activity (20 minutes): Pattern-seeking task – a real experience of Gauss**

Engage Student Teachers in the following task, which will enable them to see the beauty of pattern-seeking and understand the mathematical rule generally used to solve similar tasks.

Pattern-seeking task: A mathematics teacher gave the class the task to sum the integers 1 to 100. Gauss, a mathematician, completed the task within a few seconds and said that the sum of the integers is 5050. How did Gauss get the answer in a few seconds?

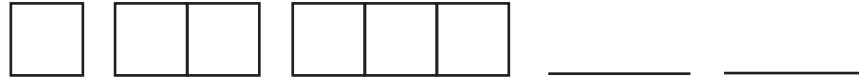
*Hint:* Gauss wrote down 1 to 10 and paired the first and last number, that is,  $1 + 10 = 11$ . There were five pairs, so 11 times 5 is 55. This means the sum of 1 to 10 integers is 55. Similarly,  $1 + 100 = 101$ ,  $2 + 99 = 101$ ,  $3 + 98 = 101$ , ...,  $50 + 51 = 101$  was written. Since there are 50 pairs of numbers, each of which adds up to 101, the sum of all the numbers must be  $50 \times 101 = 5050$ . This technique provides another way of deriving the formula, namely  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$  for the sum of the first 'n' positive integers. You need only display the consecutive integers 1 through n in two rows as follows:

1	2	3 ...	n-1	n
n	n-1	n-2 ...	2	1

Addition of the vertical columns produces n terms, each of which is equal to n+1; when these terms are added, we get the value n(n+1). Because the same sum is obtained on adding the two rows horizontally, what occurs is the formula  $n(n+1) = 2(1 + 2 + 3 + \dots + n)$ . Then dividing by 2 gives the actual formula to calculate the sum of integers.

### Activity 1 (20 minutes): Specialization leading to generalization

*Worksheet:* Use sticks to form the following pattern. Build the next two shapes.

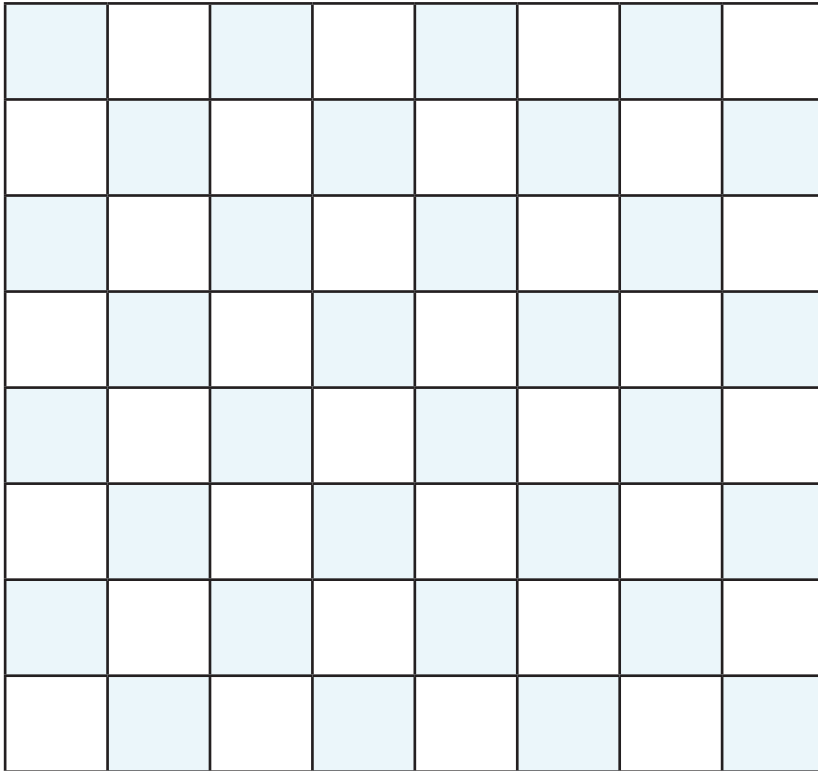


<b>No. of squares</b>	1	2	3	4	5	6	7	10	13
<b>No. of sticks</b>	4	7							

1. Fill in the grid based on the models developed with matchsticks, only up to the fifth model.
2. Predict the number of sticks needed to make 6 squares. Give a reason for your response.
3. How many sticks are needed for 10 squares?
4. Is there a possibility of having 13 sticks in any model? Give a reason for your response.
5. Is there a possibility of having 100 sticks in any model? Give a reason for your response.

**Activity 2 (20 minutes): Specialization leading to generalization**

How many squares are there on a chessboard? One person claimed that there are 204. How many can you find? Record your observation systematically. Hint: Start small, i.e. possible squares in a 1 by 1 square, then a 2 by 2 square, then a 3 by 3 square ... and fill the given table.



Square sizes	Possible number of squares
1 × 1	1
2 × 2	
3 × 3	

NOTE FOR ACTIVITY 2: If Student Teachers are not able to complete the pattern-seeking task in the session, this could be given as a home task. The following is the expected response:

Square sizes	Possible number of squares
$1 \times 1$	1
$2 \times 2$	4
$3 \times 3$	9
$4 \times 4$	16
$5 \times 5$	25
$6 \times 6$	36
$7 \times 7$	49
$8 \times 8$	64
<b>Total</b>	<b>204</b>



## Week 2: Pattern-seeking in mathematics

### Session 1: Pattern-seeking (cont.) (1 hour)

#### Activity (20 minutes): Exploring a number grid

Give Student Teachers the following number grid and ask them to share what mathematical concepts can be explored using the grid.

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>
<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>
<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>
<b>41</b>	<b>42</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>	<b>50</b>
<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>	<b>57</b>	<b>58</b>	<b>59</b>	<b>60</b>
<b>61</b>	<b>62</b>	<b>63</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>	<b>70</b>
<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>	<b>78</b>	<b>79</b>	<b>80</b>
<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>	<b>90</b>
<b>91</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>	<b>99</b>	<b>100</b>

The following are the expected responses:

- Times table
- Even and odd numbers
- Generalization
  - Even + even = ?
  - Even + odd = ?
  - Even × even = ?
  - Odd × odd = ?
- Prime and composite numbers

Refer to this website for teaching mathematical concepts using a 1 to 100 number grid:

➤ <http://guidedmath.wordpress.com/2010/08/24/25-things-to-do-with-the-hundreds-grid-in-a-guided-math-group-or-math-center/>

### **Activity (10 minutes): The importance of pattern-seeking in mathematics**

- Learning and sharing the following guidelines for conducting pattern-seeking tasks
- Give clear instructions and appropriate time to work on the task.
- Prepare worksheets for pictorial, concrete, or numerical models with appropriate instructions and provide enough space to present their working.
- If providing concrete models, draw at least two to three models on a board to guide learners to construct the desired models.
- Discuss the following steps with learners:
  - Step 1: Break down the problem into simple stages.
  - Step 2: Set up a table of results.
  - Step 3: Predict the rule after a few entries.
  - Step 4: Test the rule.
  - Step 5: Use your rule to complete the remaining tasks.

### **Activity (30 minutes): National Curriculum for Mathematics (2006)**

Divide Student Teachers into four groups, and ask them to read thoroughly the learning expectations and identify the topics or concepts that can be explored through pattern-seeking.

Group 1: Grades 1 and 2

Group 2: Grades 3 and 4

Group 3: Grades 5 and 6

Group 4: Grades 7 and 8

## Session 2: 2006 National Curriculum for Mathematics (cont.) (1 hour)

### Activity (60 minutes)

Ask groups to present their work to the whole class, obtain feedback, and finalize the list for further reference.

NOTE: Use the following table as reference to support Student Teachers while they are identifying the concepts to be taught by pattern-seeking.

Grade	Concepts and topics
III	• Divisibility rule
	• Fraction addition and subtraction with common denominators
	• Equivalent fractions
	• Odd and even numbers
IV	• Divisibility rule
	• Fraction multiplication with whole numbers
	• Area of a square and a rectangle
	• Perimeter of a square and a rectangle
V	• Fraction multiplication and division
	• Area of a triangle
VI	• Area of a parallelogram
	• Volume of cube or cuboid
	• Algebraic expression
VII	• Integer multiplication
	• Laws of exponent
	• Circumference of a circle
	• Area of a circle
	• The concept of root
VIII	• Ratio and proportion
	• Laws of exponents
	• Terminating and non-terminating numbers
	• Pythagoras theorem
	• Algebraic identities

## Session 3: Lesson-planning for teaching mathematics concepts through pattern-seeking (1 hour)

### Activity 1 (30 minutes)

Ask Student Teachers to identify a pair teacher with whom to work in developing lesson plans for any mathematical concept that can be taught using pattern-seeking as a key strategy.



### **Activity 2 (30 minutes): Micro-teaching**

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In their respective groups in which they reviewed the National Curriculum for Mathematics, ask Student Teachers to share the lesson plan and teach pattern-seeking activity to explore mathematical concepts.

### **Home task**

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Ask Student Teachers to review another pair's lesson plan and give feedback. After they have incorporated the feedback, let them compile the final version and photocopy a few for future reference.

## Week 3: Developing problem-solving skills



### **Faculty preparation**

Read 'Mathematical Framework – Problem solving' from pp. 5 to 9 of the mathematics curriculum of Singapore, retrieved from <http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf>. This shows that through a problem-solving approach, students' mathematical concepts, skills, processes, attitudes, and meta-cognition can be developed. Student Teachers can also be referred to this website.

Before the session, also read Tripathi, P. N. (n.d.). *Problem solving in mathematics: A tool for cognitive development*. Retrieved from [http://cvs.gnowledge.org/episteme3/pro\\_pdfs/27-tripathi.pdf](http://cvs.gnowledge.org/episteme3/pro_pdfs/27-tripathi.pdf). In this paper, Tripathi tries to answer the three key questions: How can problem-solving be used as a tool for cognitive development? Can problem-solving be used to effect a change in learners' attitudes and beliefs about mathematics, so that they come to view it as a discipline founded on reasoning? What are some strategies that teachers may use in the process?

### **Session 1 (1 hour): Problem-solving**

#### **Activity 1 (30 minutes): Discount or income tax**

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Engage Student Teachers in the following problem-solving task, first individually and then in a group discussion, with an instruction to record the process and the steps they used to solve the task.

**Task:** In a warehouse, you obtain a 20% discount but you must pay a 15% sales tax. Which will you prefer to have calculated first, the discount or the tax?

Ask Student Teachers to present their work on the given task. Summarize the discussion with the framework below.

The four steps to solve the problem-solving task are as follows:

**1. Find out**

Look at the problem.

Have you seen a similar problem before?

If so, how is this problem similar? How is it different?

What facts do you have?

What do you know that is not stated in the problem?

**2. Choose a strategy**

How have you solved similar problems in the past?

What strategies do you know?

Try a strategy that seems as if it will work.

If it doesn't, it may lead you to one that will.

**3. Solve it**

Use the strategy you selected and work the problem.

**4. Look back**

Reread the question.

Did you answer the question asked?

Is your answer in the correct units?

Does your answer seem reasonable?

**Activity 2 (30 minutes)**

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Engage Student Teachers in solving the following problem-solving tasks using different strategies. It is not necessary to give the strategy to use in solving the task. Summarize at the end of the session.

**1. Find a pattern**

**Task:** Anam has written a number pattern that begins with 2, 4, 6, 8, and 10. If she continues this pattern, what are the next four numbers in her pattern?

**2. Make a table**

**Task:** You save Rs. 3 on Monday. On the next day, you save the double the previous day's saving. If this pattern continues, how much will you save on Friday?

**3. Working backwards**

**Task:** Jamil walked from school A to school B in 1 hour 25 minutes. Then he took 25 minutes to walk from school B to school C. He arrived at 2:45 p.m. at school C. At what time did he leave school A?

## Session 2 (1 hour): Problem-solving strategies

### Activity (5 minutes)

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Discuss the self-assessment checklist (see Readings and Resources) for Student Teachers to monitor their own progress on the given problem-solving task, even though you will conduct an informal assessment while Student Teachers are working on the task.

### Problem-solving task (20 minutes): Handshake

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Fill in the self-assessment checklist while solving the following handshake problem:

You are in a room with 35 people. Everyone is asked to shake hands with everyone else. How many handshakes will there be? How can you figure this out? What strategies will you use?

### Problem-solving tasks (35 minutes)

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#### 4. *Guess and check*

**Task:** Azam and Zahida sold 12 show tickets altogether. Azam sold 2 more tickets than Zahida. How many tickets did each sell?

#### 5. *Draw a picture*

**Task:** Laila has 3 green balls, 4 blue balls, and 1 red ball in her bag. What fractional part of the bag of balls is green?

#### 6. *Make a list*

**Task:** Salma is taking pictures of Alina, Karim, and Mustafa. She asks them, 'How many different ways could you three children stand in a line?'

#### 7. *Write a number sentence*

**Task:** Sameer put 18 pencils in 3 equal groups. How many pencils are in each group?

#### 8. *Compute or simplify*

Many problems are straightforward and require nothing more than the application of arithmetic rules. When solving problems, simply apply the rules and remember the order of operations.

**Task:** Given  $(63)(54) = (N)(900)$ , find N.

#### 9. *Use a formula*

Formula are one of the most powerful mathematical tools at our disposal. Often, the solution to a problem involves substituting values into a formula or selecting the proper formula to use. When students encounter problems for which they do not know an appropriate formula, they should be encouraged to discover the formula for themselves.

**Task:** How many degrees are in the Celsius equivalent of  $-22^{\circ}\text{F}$ ?

### **10. Make a model**

Mathematics is a way of modelling the real world. A mathematical model has traditionally been a form of an equation. Physical models are often useful in solving problems. There may be several models appropriate for a given problem. The choice of a particular model is often related to the knowledge and problem-solving experience. Objects and drawings can help you to visualize problem situations. Acting out is also a way to visualize the problem. Writing an equation is an abstract way of modelling a problem situation. The use of modelling provides a method for organizing information that could lead to the selection of another problem-solving strategy.

**Task:** Four holes are drilled in a straight line in a rectangular steel plate. The distance between hole 1 and hole 4 is 35 mm. The distance between hole 2 and hole 3 is twice the distance between hole 1 and hole 2. The distance between hole 3 and hole 4 is the same as the distance between hole 2 and hole 3. What is the distance, in millimetres, between hole 1 and hole 3?

### **11. Act out the problem**

There may be times when you experience difficulty in visualizing a problem or the procedure necessary for its solution. In such cases, you may find it helpful to physically act out the problem situation. You might use people or objects exactly as described in the problem or you might use items that represent the people or objects. Acting out the problem may itself lead you to the answer or it may lead you to another strategy that will help you find the answer. Acting out the problem is a strategy that is very effective for young children.

**Task:** There are five people in a room and each person shakes every other person's hand exactly once. How many handshakes will there be?

### **12. Consider a simpler case**

The problem-solving strategy of simplifying is most often used in conjunction with other strategies. Writing a simpler problem is one way of simplifying the problem-solving process. Rewording the problem, using smaller numbers, or using a more familiar problem setting may lead to an understanding of the solution strategy to be used. Many problems may be divided into simpler problems to be combined to yield a solution. Some problems can be made simpler by working backwards. Sometimes a problem is too complex to solve in one step. When this happens, it is often useful to simplify the problem by dividing it into cases and solving each one separately.

**Task:** How many possible squares are on a chessboard (i.e. an  $8 \times 8$  square board)?

### **13. Eliminate**

People in everyday life commonly use the strategy of elimination. In a problem-solving context, we must list and then eliminate possible solutions based upon information presented in the problem. The act of selecting a problem-solving strategy is an example of the elimination process. Logical reasoning is a problem-solving strategy that is used in all problem-solving situations. It can result in the elimination of incorrect answers, particularly in 'if-then' situations and in problems with a list of possible solutions.

**Task:** What is the largest two-digit number that is divisible by 3 whose digits differ by 2?

The following are 13 problem-solving strategies discussed in this course.:

1. Find a pattern
2. Make a table
3. Work backwards
4. Guess and check
5. Draw a picture
6. Make a list
7. Write a number sentence
8. Compute or simplify
9. Use a formula
10. Make a model
11. Act out the problem
12. Consider a simpler case
13. Eliminate

### Session 3: National Curriculum for Mathematics (2006) (1 hour)

#### **Activity (60 minutes): National Curriculum for Mathematics (2006)** **suggested teaching strategies**

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Ask Student Teachers to read the National Curriculum for Mathematics teaching strategies (see Readings and Resources) available on the Ministry of Education website. Generate a detailed discussion to understand the role of the teacher in implementing the National Curriculum. What are the suggested approaches to be used to engage students in learning mathematics in general and problem-solving in particular? Then, ask Student Teachers to do a group task reviewing the National Curriculum for one or two grades and identify the concept that can be introduced by the problem-solving task. Design at least one problem-solving task for students to explore or apply the concept learnt in the mathematics classroom. Arrange a group presentation for Student Teachers to share and exchange ideas with colleagues, and then let them compile the ideas and problem-solving tasks for future reference.

## Ideas on understanding teachers' belief systems

Rethinking mathematical learning processes and exploring belief systems about mathematics are important for Student Teachers as they prepare themselves to become effective mathematics teachers. They will enter the mathematics class with many memories of learning mathematics at different levels of schooling; therefore, before beginning the unit on the processes of mathematics, carry out activities on rethinking mathematics as a subject. Ask Student Teachers to recall memories of learning mathematics and ask them to record these memories in their notebook.

### Task 1: Recollecting memories of learning mathematics

Review your most vivid memories about learning mathematics. In particular, try to recall the following:

- Your own learning of mathematics as a pupil in school: What were your perceptions and emotions as you participated in those lessons? Does any specific classroom incident come to mind?
- Your experiences of learning mathematics outside the classroom
- Your observation of children learning mathematics: Try to recall a moment when you were aware of particular learners and their engagement, or lack of engagement, with mathematics.

Make brief notes in your notebook that will help you recall any of these incidents.

### Discussion

The following case will help elaborate on how mathematics experiences can be automated.

#### **A case study on automation by Anam (Pseudonym)**

I recall how things frequently practiced in early childhood could become automated without one realizing it. 'As a primary schoolchild I remember getting penance. One of them was in arithmetic. We had a school inspection and the inspectors asked me how many yards make one mile. Of course I had to do some calculations but I was not given the time and I fumbled. That evening I had to attend a penance class after school and had to write 1000 times, '1760 yards make one mile'. Did I know how I got to this answer? No, but the correct answer is automated now, just as I learnt tables off by heart. This habit has stuck with me and I do it subconsciously now.'

## Task 2: Sentence completion

Sometimes probes are useful in reflecting on how we feel, view, and appreciate experiences. Ask Student Teachers to complete the following statements to describe their feelings about mathematics:

- Learning mathematics is like \_\_\_\_\_
- I enjoy learning mathematics when \_\_\_\_\_
- I dislike learning mathematics when \_\_\_\_\_

Hold a discussion on common and not-so-common statements. You can also use the data to prepare a graph on Student Teachers' feelings and measure if there are more positive or negative feelings. You can use this data to explore why a fear of mathematics, or a lack thereof, is established.

## Task 3: Using metaphors and analogies

Analogies and metaphors are also great motivators and introductory activities for reflection. Ask Student Teachers to offer their personal metaphors and images, such as:

- Learning mathematics is like \_\_\_\_\_
- using common sense
- climbing a mountain
- learning the rules of the game
- peeling an onion
- using a cookbook.

## Task 4: Describing teaching

Now try to write down some of your beliefs about teaching mathematics. Complete the following clues:

- Teaching mathematics is like \_\_\_\_\_
- What I like most about teaching mathematics is \_\_\_\_\_
- What I like least about teaching mathematics is \_\_\_\_\_

## Discussion

As with learning, metaphors for teaching may be very personal and may vary, even for the same individual at different times.

- Teaching mathematics is like \_\_\_\_\_
- learning something new every day
- pushing water uphill (sometimes downhill)
- banging one's head against a wall
- juggling balls in the air
- opening up new vistas for pupils
- a roller coaster.

Completed positive clues could be:

- a sense of achievement (aha!!!)
- when a difficult topic is seen by pupils as easy
- seeing pupils enjoy themselves
- the sense of fulfilment when a problem 'clicks' for a pupil.

## Task 5: Student Teachers' beliefs about the nature of mathematics

Previous activities help you rethink your beliefs about teaching and learning mathematics. But what are your beliefs about the nature of mathematics itself? One of the key ways in which your perception of the nature of mathematics has developed is through your own experience of learning and doing mathematics. This experience will also have a bearing on your understanding of how mathematics is learned and on your perceptions of the roles of teachers and pupils in mathematics classrooms. You may also experience strong feelings and emotions relating to your own work on mathematical tasks.

Read and complete the given statements:

- Mathematical ideas come from \_\_\_\_\_
- Mathematics is important in schools because \_\_\_\_\_
- Mathematics is the mother of all disciplines because \_\_\_\_\_

Student Teachers may respond that ideas come from people, books, television, daily life, and nature. They may say that it is a scoring subject, or that it is seen as a measure of academic ability, or that mathematically smart people are intelligent people. Some may say it helps people think logically, and it helps solve problems, for example. Collate all responses and see if Student Teachers see mathematics as content or as a process. By doing these tasks, you will set a base for them to build their understanding of why different pedagogies are required to teach mathematics and not just automated practices.



## Ideas on ICT integration in mathematics learning

Generally Student Teachers have different views on the use of technology in a mathematics classroom. Before embarking on the discussion, engage them in a debate about the topic 'Calculators hamper or facilitate students' learning' to explore their perception of mathematics. Put Student Teachers into groups based on whether they agree with the statement. Note the key points while they are presenting their argument to justify their stance. Then generate a discussion to help them understand that it depends on teachers at what point in time they use the technology. For instance, a calculator may be an appropriate tool for dealing with complicated and tedious calculations with large numbers (for example, how many seconds there are in 15 hours). Nevertheless, students should be competent in making sense of the answer and need to possess estimation skills. For instance, if students can estimate the answer when dividing a decimal number by another decimal number, the answer displayed on the calculator screen will make sense to them.

An analogy of building construction can be used to understand the importance of a foundation in learning mathematics. A high-rise building can never stand without a strong foundation. Similarly, basic computation understanding and skills are a foundation for mathematics learning. Then technology is used to make students more confident mathematics learners, so that they will be able to take on challenging tasks. This means that children need to understand the basics first, after which they can be engaged in using technology to build further mathematical learning. Thus, the use of a calculator in no way reduces the need for mathematical understanding on the part of the individual who is using it.

The following are some of the ideas to engage Student Teachers in using calculators in the mathematics classroom:

- Give Student Teachers a target number that they have to get on the calculator, but choose several keys that are 'broken' and cannot be used. For instance, tell them to obtain 0.2525 without using 2, 5, or the + key.
- Give Student Teachers a large number to enter, such as 79348. Then tell them to subtract a number to leave a 0 in the hundreds column. (They would have to subtract 300.) And so forth.
- Pick a fairly large number at the start, such as 2345753. Then pick a range of numbers, such as 1 to 9. Players take turns in dividing the large number by a number in the range. The player who reaches a single-digit number first is the winner.
- Many variations can be tried in a grocery store. One idea is to give Student Teachers a list of items to 'purchase'. They are allowed to buy up to four of any one item. With two or more Student Teachers, whoever gets closest to a total cost without exceeding it wins. If there is just one Student Teacher, give him/her a target amount to spend and see how close s/he can get.

Cabri software can be used to develop geometry concepts and generalizations. The task sheet (see Readings and Resources) will help you to take Student Teachers through a process of exploring basic concepts of geometry using Cabri.

4

# Readings and resources



This section contains selected readings that faculty will find useful in preparing for sessions, as well as student readings. Teaching materials, such as handouts or activities that require elaboration, are also included. In some cases, original pieces written specifically for this course are included. The readings in the section Annotated Bibliography are used with permission of the author or publisher. Therefore, these are for classroom use and may be duplicated for distribution to Student Teachers. Intellectual property rights are respected throughout. Those who wish to use these readings in their own publications will need to consider intellectual property rights, secure permission for their use, and include proper attribution to sources.

Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A survey of secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research*, 32(2), 45–61. Retrieved from:

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5

# Annotated bibliography



# Developing and deepening mathematical knowledge in teaching: Being and knowing

**By Anne Watson**

Watson, A. *Developing and deepening mathematical knowledge in teaching: Being and knowing*. Retrieved from:

➤ [http://www.maths-ed.org.uk/mkit/MKiT5\\_Watson\\_distribution-version.pdf](http://www.maths-ed.org.uk/mkit/MKiT5_Watson_distribution-version.pdf)

In this paper, Anne Watson tries to think about mathematical knowledge in teaching as a way of being and acting, avoiding categorization and acquisition metaphors of knowledge. She thinks of it as participation in mathematical practices in the classroom and also during preparation for teaching. Thus the development and deepening of knowledge take place through doing mathematics and being mathematical in social contexts in which mathematical habits of mind are embedded, recognized, and valued. She explains how some of the tasks of teaching can be seen as particular contextual applications of mathematical modes of enquiry. She argues that experience of doing mathematics, on one's own and with others, in an environment that encourages listening, questioning, and pedagogic reflection (which may be the teacher's own classroom), develops and deepens mathematical knowledge both in and for teaching.

Anne Watson writes the following summary in the paper (pp. 7–8):

Further experience of learning mathematics, rather than learning a pedagogic view of mathematics, is a good way to deepen and develop mathematical knowledge in and for teaching. I have supported this conclusion by:

- Arguing against the use of typographies which are generated by a pedagogic view
- By offering examples of curriculum difficulties which can be understood by analysing their mathematical components and affordances
- Arguing that understanding the activity of doing mathematics provides all the analytical tools needed to think about learning and teaching, and other people's mathematical activity
- Recognising that inherent difficulties in mathematics can be found by a mathematical analysis which avoids pathologising learners and teachers
- By giving examples of teachers who are non-specialists and showing that what they do not do is dependent on personal knowledge; on the other hand, a teacher with a similar background revealed that he still was an active mathematician within his own sphere of interest
- By giving examples of teachers who studied more maths deliberately to enhance their teaching, and showing how their new learning impacted on teaching
- By showing how past and current experience as a learner can inform teaching.

She also indicated that there was no need to imagine that learning more mathematics while teaching needs to be confined to engagement in courses, but that mentors in school can create opportunities for such learning. She said that, 'I am not claiming to have presented a fully joined-up robust argument that it is engagement in the practices of mathematics that enables good teaching, but I do claim to have offered some directions' (p. 8).

## How secondary teachers structure the subject matter of mathematics

**By Anne Watson**

Watson, A. (2008). How secondary teachers structure the subject matter of mathematics. In M. Joubert, (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 28(3). Retrieved from:

➤ <http://www.bsrlm.org.uk/IPs/ip28-3/BSRLM-IP-28-3-22.pdf>

In this paper, based on analysing 40 lesson videos available from various studies, Anne Watson shares that they have come to understand that example choice, task design, variation, and certainly key mathematical activities play a part in engagement and learning, whatever the teaching style, social context, lesson structure, and interaction patterns.

## School mathematics as a special kind of mathematics

**By Anne Watson**

Watson, A. *School mathematics as a special kind of mathematics*. Available from:

➤ <http://www.unige.ch/math/EnsMath/Rome2008/WG1/Papers/WATSON.pdf>

Anne Watson writes the following:

In this paper I argue that school mathematics is not, and perhaps never can be, a subset of the recognized discipline of mathematics, because it has different warrants for truth, different forms of reasoning, different core activities, different purposes, and necessarily truncates mathematical activity. In its worst form, it is often a form of cognitive bullying which neither develops students' natural ways of thinking in advantageous ways, nor leads obviously towards competence in pure or applied mathematics as practised by adult experts. The relationship of school mathematics to adult competence is similar to the relationship between doing military drill and military leadership; between being made to eat all your spinach and becoming a chef; between being forced to practise scales and becoming a pianist. There are some connections, but they are about having a focus on a narrow subset of semi-fluent expertise in negative social and emotional contexts, without full purpose, context and meaning. That some people become effective military leaders, beautiful pianists or inspiring cooks is interesting, but what is more interesting is the fact that most people who go through these early experiences do not: instead they merely



follow orders, or hate green vegetables, or give up practising their instruments. The image around which I hang this paper is the cafeteria at the heart of the mathematics faculty at Cambridge. It is a large room with coffee at one end, small tables covered with papers and laptops, each surrounded by four chairs, students (mainly male), some undergraduate, some graduate, pure and applied, some alone, some in casual groups, some in self-organised study groups with their own internal disciplines and plans, taking time, talking, arguing. There are many furrowed brows, people leaning forward with arms and hands working away to express some thought; bodies, words and minds hauling together to communicate how they see relationships and properties, and offering each other handles to work with abstract objects and multi-dimensional extensions. No one is doing exercises or practising techniques; no one is interrupted by bells or instructions to change to the next task; no one is taking their work to teachers to be marked. Every now and then there are whoops of excitement or groans of frustrated realisation.

She further writes that:

‘Doing mathematics’ is predominantly about empirical exploration, logical deduction, seeking variance and invariance, selecting or devising representations, exemplification, observing extreme cases, conjecturing, seeking relationships, verification, reification, formalisation, locating isomorphisms, reflecting on answers as raw material for further conjecture, comparing argumentations for accuracy, validity, insight, efficiency and power. It is also about reworking to find errors in technical accuracy, and errors in argument, and looking actively for counterexamples and refutations. It can also be about creating methods of problem-presentation and solution for particular purposes, and it also involves, after all this, proving theorems.

## Ongoing impact of the “advanced diploma in education: Mathematics”

By **Dr Anjum Halai, Dr Munira Amirali, Nadeem Kirmani, and Dr Razia Fakir Muhammad**

Halai, A., Ali, M. A., Kirmani, N., & Muhammad, R. F. (2003). Ongoing impact of the “advanced diploma in education: Mathematics”. *Impact: Making a difference*, 135–146. Retrieved from:

➤ [http://ecommons.aku.edu/cgi/viewcontent.cgi?article=1053&context=book\\_chapters](http://ecommons.aku.edu/cgi/viewcontent.cgi?article=1053&context=book_chapters)

This paper presents the findings of the action research project in mathematics education, which was undertaken to study the impact on teaching and learning of strategies introduced in the Advanced Diploma Programme in Education: Mathematics (2003), offered by the Aga Khan University Institute for Educational Development. Findings from the study showed that the teaching strategies introduced as part of the advanced diploma programme led to a positive change in the teachers’ classroom practice, in the mathematics that the students learnt in the

classroom, and in how they learnt it. However, the study also revealed that certain contextual and other factors mediated the potential of these strategies to influence the classroom. The findings reported here primarily discuss the teachers' subject matter knowledge, their experience and ability to handle student responses, and their critical (as opposed to an unquestioning) use of the strategies introduced.

## Students' conceptions of the nature of mathematics and attitudes towards mathematics learning

By Dr Munira Amirali

Amirali, M. (2010). Students' conceptions of the nature of mathematics and attitudes towards mathematics learning. *Journal of Research and Reflections in Education*, 4(1), 27–41. Retrieved from:

➤ <http://www.ue.edu.pk/jrre/articles/41003.pdf>

**Abstract:** Students' conceptions of the nature of mathematics have a great influence towards mathematics learning. In order to understand gender-wise students' conceptions and attitudes towards mathematics this study has been conducted in Karachi, Pakistan. This paper examines patterns in students' conception and attitude towards mathematics by analyzing survey data obtained from 82 students studying in grade eight in a private school context. The survey was conducted using a five-point Likert scale ranging from 'strongly agree' through 'neutral' to 'strongly disagree'. The survey findings illustrate that students consider mathematics as a useful subject which is used in daily life routines and facilitates in developing problem-solving skills and to strengthen future career. However, the findings also highlight students' confusion and contradictions in terms of the nature of mathematical knowledge i.e., they show their level of agreement to both 'absolutist' and 'fallibilist' view of mathematics. With respect to mathematics attitude the results shows that female students hold a more positive attitude towards mathematics and lesser mathematical anxiety than their male counterparts.

# Effective pedagogy in mathematics

By Glenda Anthony and Margaret Walshaw

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. International Bureau of Education, UNESCO. Available from

➤ [http://www.ibe.unesco.org/fileadmin/user\\_upload/Publications/Educational\\_Practices/EdPractices\\_19.pdf](http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf)

This booklet focuses on effective mathematics teaching. Drawing on a wide range of research, it describes the kinds of pedagogical approaches that engage learners and lead to desirable outcomes. The aim of the booklet is to deepen the understanding of practitioners, teacher educators, and policymakers and assist them to optimize opportunities for mathematics learners.

Mathematics is the most international of all curriculum subjects, and mathematical understanding influences decision-making in all areas of life – private, social, and civil. Mathematics education is a key to increasing the post-school and citizenship opportunities of young people, but today, as in the past, many students struggle with mathematics and become disaffected, as they continually encounter obstacles to engagement. It is imperative, therefore, that we understand what effective mathematics teaching looks like and what teachers can do to break this pattern. The principles outlined in this booklet are not stand-alone indicators of best practice: any practice must be understood as nested within a larger network that includes the school, home, community, and wider education system. Teachers will find that some practices are more applicable to their local circumstances than others. Collectively, the principles found in this booklet are informed by a belief that mathematics pedagogy must:

- be grounded in the general premise that all students have the right to access education and the specific premise that all have the right to access mathematical culture
- acknowledge that all students, irrespective of age, can develop positive mathematical identities and become powerful mathematical learners
- be based on interpersonal respect and sensitivity and be responsive to the multiplicity of cultural heritages, thinking processes, and realities typically found in our classrooms
- be focused on optimising a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning
- be committed to enhancing a range of social outcomes within the mathematics classroom that will contribute to the holistic development of students for productive citizenship.
-

6

# Handouts



## Pre-test



This pre-test was developed by Dr. Munira Amirali and permission was granted for its use in the Mathematics III course. For educational purposes only.

**Below are some generalized statements or rules. Write a reason(s) for each of the given statements or rules, with an example.**

1. When two fractions are multiplied, we multiply the numerator by the numerator and the denominator by the denominator.
2. When two fractions are divided, we inverse the divisor fraction (write it upside down) and then multiply the numerator by the numerator and the denominator by the denominator.
3. When a positive number is multiplied by a negative number, the resultant number is negative.
4. When a negative number is multiplied by a negative number, the resultant number is positive.
5. When two decimal numbers are multiplied, first multiply the given numbers by removing the decimal points, then add the number of places after the decimal points, and then put the decimal point starting from the left (for example:  $1.2 \times 2.6$  as  $12 \times 26 = 312$ ,  $3.12$ )
6. Why is the area of circle  $\pi r^2$ ?



## Discovering the area formula for circles

Using a circle that has been divided into congruent sectors, students will discover the area formula by using their knowledge of parallelograms. Students will then calculate the area of various flat circular objects that they have brought to school. Finally, students will investigate various strategies for estimating the area of circles.

### Learning objectives

Students will:

- Measure the radius and diameter of various circular objects using appropriate units of measurement
- Discover the formula for the area of a circle
- Estimate the area of circles using alternative methods

### Materials

Circular objects

Calculators

Scissors

Compasses

Rulers

'Area of Circles' activity sheet (<http://illuminations.nctm.org/Lessons/ApplePi/ApplePi-AS-Circles.pdf>)

'Fraction Circles' activity sheet (<http://illuminations.nctm.org/Lessons/ApplePi/ApplePi-AS-FractionCircles.pdf>)

Centimeter grid paper on overhead transparencies

Blank copy paper

### Instructional plan

Prior to the lesson, ask students to bring in several flat, circular objects that they wish to measure with their classmates.

As a warm-up, give students an opportunity to estimate the area of the circular objects that they have brought to class. Working in groups and using the Area of Circles activity sheet, students should **individually** complete the first two columns:

- Description of the object
- Their estimate for the area of the object

(The other two columns will be completed later in the lesson.)

## Area of circle

Ask students to identify several flat, circular objects that they wish to measure.

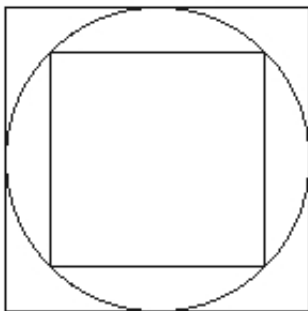
Give students an opportunity to estimate the area of the circular objects that they have brought to class. Working in groups and using the table below, students should individually complete the first two columns:

- Description of the object
- Their estimate for the area of the object

Description of the Object	Your estimate of the area (in square centimeters)	Radius of the object	Actual Area

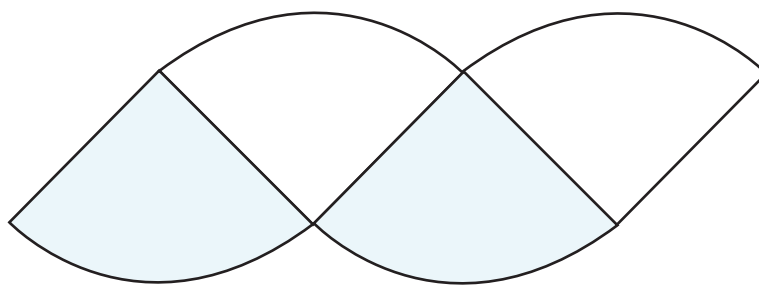
Students may use any method they like to estimate the area of their objects. Some possible methods include:

- Students can trace the shape of their object on a piece of centimeter grid paper and count how many square centimeters make up the total area of the circle.
- Students can divide the circle into wedges by drawing various radii. They can approximate the area of each wedge using the triangle formula. This method is similar to a method used by Archimedes, and it is the method that will be used later in this lesson. For a connection to mathematical history, you may want to include a brief overview of Archimedes and his method for calculating the area of a circle. For more information go to:  
➤ <http://www.ugrad.math.ubc.ca/coursedoc/math101/notes/integration/archimedes.html>
- Students can inscribe the circle in a square, hexagon, or some other polygon. Then, the same shape could be inscribed within the circle. Students could determine the area of the inscribed and circumscribed shapes to get lower and upper estimates, respectively. (You may need to provide a sample drawing of this method, like the one shown below.)



After students have estimated the area of several objects, allow them to physically discover the area formula of a circle. Since this is a whole-class activity, you may wish to enlarge the manipulatives and display them on the chalkboard, or you can use them on the overhead projector.

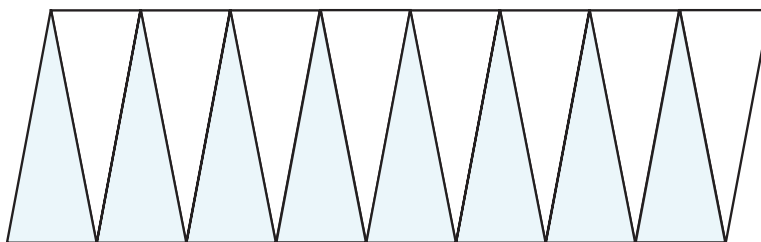
Have students cut the circle from the sheet and divide it into four wedges. (This can be done if students cut only along the solid black lines.) Then, have students arrange the shapes so that the points of the wedges alternately point up and down, as shown below:



Ask, “When arranged in this way, do the pieces look like any shape you know?” Students will likely suggest that the shape is unfamiliar.

Then, have students divide each wedge into two thinner wedges so that there are eight wedges total. (This can be done if students cut only along the thicker dashed lines.) Again, have students arrange the shapes alternately up and down. Again ask if this arrangement looks like a shape they know. This time, students will be more likely to suggest that the arrangement looks a little like a parallelogram.

Finally, have students divide each wedge into two thinner wedges so that there are sixteen wedges total. (This can be done if students cut along all of the dashed lines.) Allow students to arrange the wedges so that they alternately point up and down, as shown below:



Ask, “When the circle is divided into wedges and arranged like this, does it look like another shape you know? What do you think would happen if we kept dividing the wedges and arranging them like this?” Lead the discussion so students realize the shape currently resembles a parallelogram, but as it is continually divided, it will more closely resemble a rectangle.

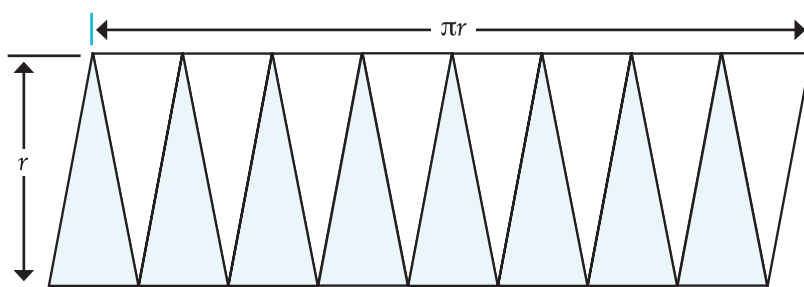
You may wish to continue this activity by having students divide the wedges even further.



Ask students, “What are the dimensions of the rectangle that is formed?” From the Circumference lesson (at <http://illuminations.nctm.org/LessonDetail.aspx?ID=L573>), students should realize that the length of the rectangle is equal to half the circumference of the circle, or  $\pi r$ . Additionally, it should be obvious that the height of this rectangle is equal to the radius of the circle,  $r$ . Consequently, the area of this rectangle is  $\pi r \times r = \pi r^2$ . Because this rectangle is equal in area to the original circle, this activity gives the area formula for a circle:

$$A = \pi r^2$$

The figure below shows how the dimensions lead to the area formula.



Allow students to return to the objects for which they estimated the area at the beginning of class. They should measure the radius of each object and record it in the third column on the ‘Area of Circles’ sheet. Then, students should use the formula just discovered, calculate the actual area of each object, and record the area in the fourth column.

Once all groups have completed the measurements and calculations, a whole-class discussion and presentation should follow. On the chalkboard, the presenter for each group should record the areas for the objects. The students should compare the results of each group and discuss the accuracy of the areas found.

The class should also compare their original estimates with the actual measurements. On their recording sheets, have them highlight the objects for which their estimates were very close to their actual. Using a few sentences, have the students explain (on the recording sheet) why some estimates were closer than others.

During the class discussion, the following are some key points to highlight:

- Emphasize that 3.14 is only one approximation for  $\pi$ . Refer to the Circumference lesson, and discuss the various estimates that were found for  $\pi$  and what caused these variations. Also explain that there are other approximations, but typically 3.14 is used because it is accurate enough for most situations and it is easy to remember. If students are curious, other approximations for  $\pi$  are given on the Pi Approximation sheet at <http://illuminations.nctm.org/Lessons/ApplePi/PiApprox.pdf>
- The total area is almost always an approximation. Because the value of  $\pi$  can only be approximated, any time the area of a circle is stated without the  $\pi$  symbol, it must be an approximation. For instance, a circle with radius of 5

inches has an exact area of  $25\pi$  in.<sup>2</sup> and an approximate area of 78.54 in.<sup>2</sup>. You might wish to hold a “mock debate” with one student taking each position (yes, it’s always an exact value; no, it’s not an exact value) giving examples and reasons to justify their position.

- Students should be able to calculate radius from diameter and diameter from radius. In particular, students should realize that  $d = 2r$ .
- Students should understand the area formula as described in your curriculum. Slight variations are possible, so the version in your textbook, standards, or other materials may be different from the formula presented in this lesson.

## Questions for students

In your opinion, why did we use the properties of a parallelogram to discover the area formula for circles?

[Determining the area of a circle is difficult. By converting a circle to a parallelogram, we can use the formula for the area of a parallelogram to determine the area of the circle.]

When would it be necessary to know the exact area of a circle? When would an estimate be sufficient? Explain your thinking. [Student responses may vary.]

Why did we approximate our answers for area? Can the area of a circle ever be exact?

[It is not possible to find an exact numeric value for  $\pi$ . Therefore, all calculations of area must be approximations (unless the answer is left in “exact form,” which means using the symbol  $\pi$  to express the answer).]

## Assessment options

1. Students can solve the following practice problem:
  - o The radar screens used by air traffic controllers are circular. If the radius of the circle is 12 centimeters, what is the total area of the screen?

[ $A = \pi r^2$ , so the area of the radar screen is approximately  $3.14 \times 12^2 \approx 452.16$   $\text{cm}^2$ .]

2. Working in pairs or groups, have students locate manhole covers and other circles on or near the school grounds. Have students measure the diameter of these circles and then determine the area.
3. Have students explore the following links and answer the associated questions. Circulate throughout the room to ensure on-task behavior and to check for understanding.

Circumference and area of circles at:

➤ <http://www.explorellearning.com/index.cfm?method=cResource.dspView&ResourceID=206&CFID=644815&CFTOKEN=41765401>

Circles and Pi at:

➤ [http://www.learner.org/courses/learningmath/measurement/session7/part\\_b/](http://www.learner.org/courses/learningmath/measurement/session7/part_b/)

Lesson plan available from:

➤ <http://illuminations.nctm.org/LessonDetail.aspx?ID=L574>

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## Definition of mathematics

### Statement One

“Mathematics is the creation of the mind, demonstrating powerfully the mind’s capacity to seek order and pattern in the world of experience, to construct explanations and to intellectualise about this world, and to delight in challenge and in the resolution of problems posed to it by itself.”

By John Mack in *Mathematics & Teaching, Topics for the professional development of teachers*. (1994, Pg. (iii))Ed. Chick, Helen & Watson, Jane, Australian Association of Mathematics Teachers Inc. Adelaide

### Statement Two

Mathematics is an integral part of our existence. It is a powerful form of communication which enables us to represent, to interpret, to explain and to predict.

The study of mathematics involves the search for patterns and relationships, through which we are able to explore, explain, and interpret the world around us. The focus on problem solving and investigation, which has become a significant element of mathematics learning in recent years, ensures that these possibilities are realized.

We all use at least a fundamental knowledge of number, measurement, special relationships and statistics in our daily lives. At the same time, more sophisticated understandings in these areas form the basis for a myriad of vital activities in science, the humanities and the arts.

Beyond this specific value, the study of mathematics provides opportunities to develop logical reasoning and provides a powerful means of communication.

Mathematics is exciting, challenging and satisfying. It holds an inherent interest for young children, and this can be maintained through effective teaching which emphasises practical relevance.

International Education Agency of Papua New Guinea. (1999), *The IEA Curriculum: Mathematics* pp. 6

# Survey questionnaire



Items developed by Dr Munira<sup>4</sup>

## SECTION 'A'

### 1. Nature of Mathematics

**Instructions:** This section includes statements that teachers usually think about the nature of mathematics. Please read each statement carefully, before putting a tick (✓) in the appropriate box.

#	Items:	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1.	Mathematics comprises only formulae, symbols and rules.					
2.	Current mathematical knowledge will remain the same in the future.					
3.	Mathematics existed in the world even before human creation.					
4.	Mathematics is a creative subject like arts / music.					
5.	Human beings create mathematical knowledge.					
6.	The study of mathematics is suited mostly to males.					
7.	Mathematical knowledge can contribute in addressing societal issues (e.g. inequality, environmental issues)					

<sup>4</sup> Amirali, M. (2011). *Investigating mathematics teachers' knowledge about mathematics, mathematics teaching and learning in schools in Pakistan* (Unpublished doctoral dissertation). Institute for Educational Development, Aga Khan University, Karachi.



## The National Curriculum for Mathematics 2006

The National Curriculum for Mathematics 2006 is one of the most useful resources for you and Student Teachers and one that they should be familiar with before they start teaching.

The National Curriculum for Mathematics is organized around five standards:

- Number and operations
- Algebra
- Measurement and geometry
- Information handling
- Reasoning and logical thinking

Download the curriculum for grades 1–10 from:

➤ [http://www.ibe.unesco.org/curricula/pakistan/pk\\_al\\_mt\\_2006\\_eng.pdf](http://www.ibe.unesco.org/curricula/pakistan/pk_al_mt_2006_eng.pdf)

# Self-assessment checklist



Place a tick in an appropriate column to show how you were thinking before, during, and after working on the problem-solving task.

## Before

**Before you began to solve the problem, what did you do?**

1. I read the problem more than once.
2. I tried to find everything out about the problem that I could.
3. I asked myself whether I really understand what the problem is asking me.
4. I thought about what information I needed to solve this problem.
5. I asked myself whether I have ever worked a problem like this before.
6. I asked myself whether there is information in this problem that I do not need.

## During

**As you worked the problem, what did you do?**

1. I kept looking back at the problem as I worked.
2. I had to stop and rethink what I was doing and why.
3. I checked my work as I went along step by step.
4. I had to start over and do it differently.
5. I asked myself whether what I am doing is getting me closer to the answer.

## After

**After you finished working the problem, what did you do?**

1. I checked to see if all my calculations were correct.
2. I went over my work to see if it still seemed like a good way to do the problem.
3. I looked at the problem to see if my answer made sense.
4. I thought about a different way to solve the problem.
5. I tried to see if I could tell more than what the problem asked for.

Adapted from *Elementary and Middle School Mathematics* by John A. Van de Walle.



## Exploring geometry through Cabri

### Activity 1

1. Draw a line segment AB and measure it.
2. Drag point B. What do you observe?

### Activity 2

1. Draw an angle PQR and measure it. (Hint: Draw using line segments.)
2. Drag point R. What do you observe?

### Activity 3

Draw the following three triangles:

1. Right-angled triangle showing scalene triangle property

2. Isosceles triangle showing obtuse-angled triangle property

3. Equilateral triangle showing acute-angled property

### Activity 4

1. Draw a triangle ABC.
2. Measure all three interior angles and record them in the given table.
3. Drag one vertex and record the angle measurement. Repeat the process three to four times.

No.	$\angle A$	$\angle B$	$\angle C$	$\angle A + \angle B + \angle C = ?$
1				
2				
3				
4				

4. What do you conclude?



## Activity 5

1. Draw a quadrilateral ABCD.
2. Measure all four interior angles and record them in the given table.
3. Drag one vertex and record the angle measurement. Repeat the process three to four times.

No.	$\angle A$	$\angle B$	$\angle C$	$\angle D$	$\angle A + \angle B + \angle C + \angle D = ?$
1					
2					
3					
4					

4. What do you conclude?

Note: A similar process can be used to explore the interior angles' sum of a regular pentagon, hexagon, etc.

## Activity 6

1. Draw two intersecting line segments AB and CD. Measure all the angles. What do you observe?
2. Drag point A. What do you observe now?

## Activity 7

1. Draw two parallel line segments PQ and RS. Draw a transversal TU. Measure all the angles formed. What do you observe?
2. Drag point T. What do you observe now?

## Activity 8

1. Draw a circle with a diameter AB.
2. Mark a point C on the circumference. Join points CA and CB.
3. Measure angle C and record it in the given table.
4. Drag point C and then record  $\angle C$ . Repeat the process two to three times.

No.	$\angle C$
1	
2	
3	

### Activity 9

1. Draw a circle.
2. On a circumference, mark four points A, B, C, and D and join these points to form a quadrilateral ABCD.
3. Measure all four interior angles and record them in the given table.
4. Drag point A and then record the angles' measurement. Repeat the process two to three times.

No.	$\angle A$	$\angle B$	$\angle C$	$\angle D$
1				
2				
3				

5. What do you conclude?

### Activity 10

1. Draw a trapezium ABCD.
2. Mark the midpoints E and F of the two intersecting lines. Join EF.
3. Is EF parallel to AB or CD?
4. Measure all three parallel sides. What relationship you can observe?

### Activity 11

1. Draw a triangle ABC.
2. Mark the midpoints D and E of the sides AC and BC respectively. Join DE.
3. Is DE parallel to AB?
4. Measure sides AB and DE. What relationship you can observe?
5. What relationship exists between Activities 10 and 11?

### Activity 12

1. Draw a circle with centre O.
2. On a circumference, mark three points A, B, and C. Join OA, OB, AC, and BC.
3. Measure central angle AOB and inscribed angle ACB and record them. What do you observe?
4. Drag point C and now record the angles' measurement. Repeat the process two to three times.

No.	$\angle AOB$	$\angle ACB$
1		
2		
3		

### Activity 13

1. Draw an acute-angled triangle ABC. Bisect all three angles. What do you observe?
2. Drag one vertex so that the triangle becomes an obtuse-angled triangle. What do you observe?
3. Drag one vertex so that the triangle becomes a right-angled triangle. What do you observe?

Conclusion

- a. In an acute-angled triangle, angle bisectors are \_\_\_\_\_
- b. In an obtuse-angled triangle, angle bisectors are \_\_\_\_\_
- c. In a right-angled triangle, angle bisectors are \_\_\_\_\_

### Activity 14

1. Draw an acute-angled triangle PQR. Bisect all three sides. What do you observe?
2. Drag one vertex so that the triangle becomes an obtuse-angled triangle. What do you observe?
3. Drag one vertex so that the triangle becomes a right-angled triangle. What do you observe?

Conclusion

- a. In an acute-angled triangle, side bisectors are \_\_\_\_\_
- b. In an obtuse-angled triangle, side bisectors are \_\_\_\_\_
- c. In a right-angled triangle, side bisectors are \_\_\_\_\_

### Activity 15

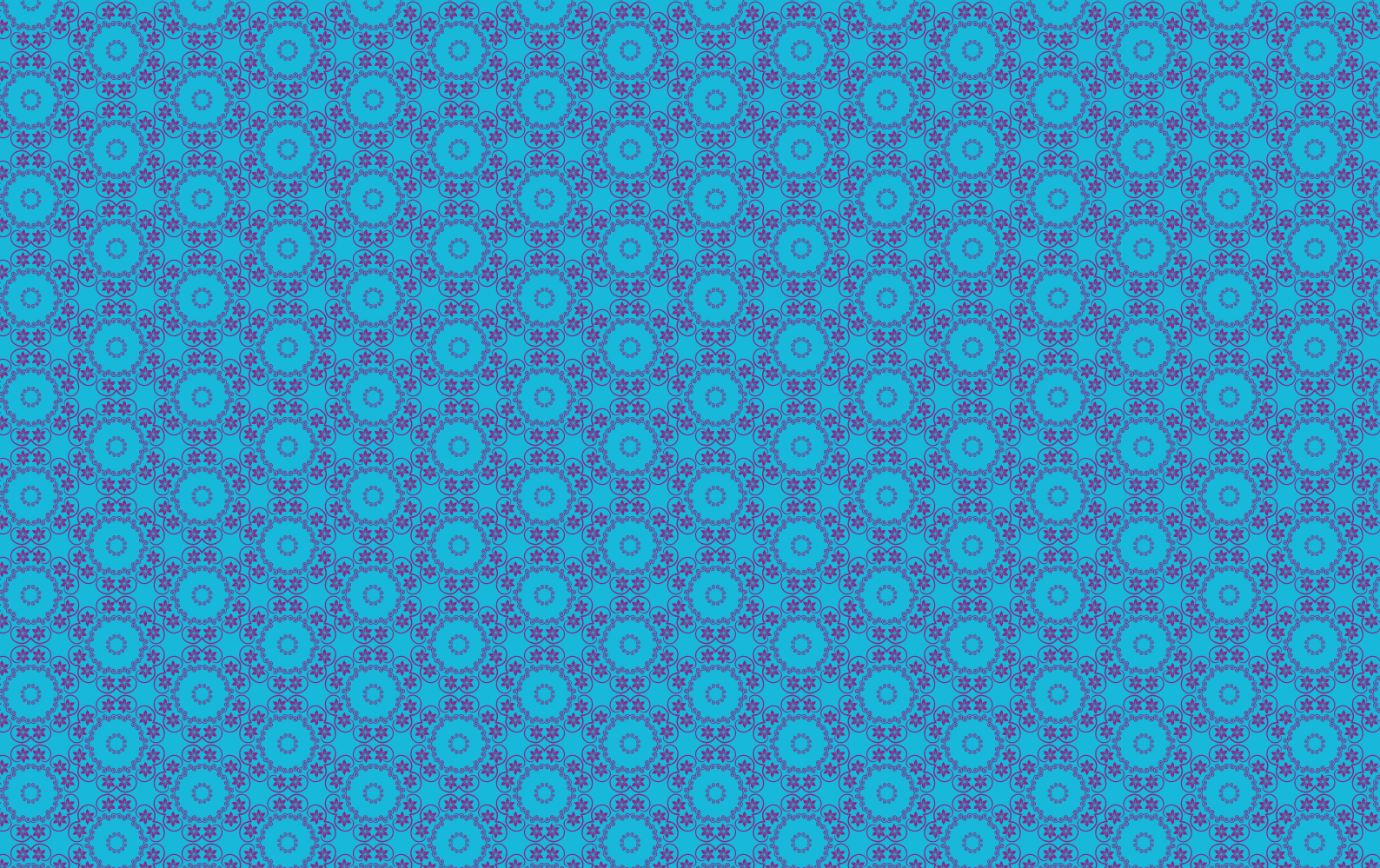
1. Draw an acute-angled triangle XYZ. Draw three altitudes from all three vertices on the side opposite to them. What do you observe?
2. Drag one vertex so that the triangle becomes an obtuse-angled triangle. What do you observe?
3. Drag one vertex so that the triangle becomes a right-angled triangle. What do you observe?

Conclusion

- a. In an acute-angled triangle, altitudes are \_\_\_\_\_
- b. In an obtuse-angled triangle, altitudes are \_\_\_\_\_
- c. In a right-angled triangle, altitudes are \_\_\_\_\_







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