Developing and deepening mathematical knowledge being and knowing in teaching

Mathematical knowledge in teaching as a way of being and acting, avoiding categorization and acquisition metaphors of knowledge. MKT as participation in mathematical practices in the classroom, and also during preparation for teaching. Thus development and deepening of knowledge take place through doing mathematics and being mathematical in social contexts in which mathematical habits of mind are embedded, recognized and valued. Some of the tasks of teaching can be seen as particular contextual applications of mathematical modes of enquiry.

However, it is *mathematical* enquiry that includes learning the traditional mathematical collections. Professional development opportunities that offer only collaborative enquiry as a remedy can be as irrelevant as those that offer only mathematical procedures. Any process of identifying types can go too far and losing overarching insight. This is why, a typographical approach to MKIT (knowledge of curriculum, knowledge of students, knowledge of textbooks, etc.) can mask the essential activity within which those nouns connect and inform each other.

Experience of doing mathematics, on one's own and with others, in an environment that encourages listening, questioning and pedagogic reflection (which may be the teacher's own classroom), develops and deepens mathematical knowledge both in and for teaching. One problem with identifying types of knowledge is that we end up with definitions which can be unhelpful for teacher educators – being too unwieldy to fit into institutional constraints– and unhelpful for learners who then get a fragmented sense of what is relevant without yet having the practical perspective to make sense of it.

Teaching as a contextual application of mathematical modes of enquiry

Teachers can learn new solutions, new methods, new properties, new distinctions and new questions by listening to learners. Sometimes, teachers express these as *not* new, they knew the mathematics already, but they came to *understand* it differently through teaching and they do not necessarily recognize this as 'learning'. Thus they learn mathematics while teaching, and the learning is embedded in the practices of teaching, and is hence 'mathematics in teaching'.

A teacher who works mathematically can identify the difficult bits of math, the bits where previous understanding is unhelpful, the places where significant new ways to understand have to be worked on with effort. In this way, the inaccurate statements of learners are seen as alternative conceptions generated by mathematical thinking: pattern-seeking, generalizing, interpreting, applying. How to engage critically with mathematics, communicated in various ways, leads to understanding of how ‘errors’ are made and therefore reduces the need to learn about individual errors. Furthermore, this approach avoids the declaration of a ‘deficiency’ model of learners and replaces it with an understanding that epistemological obstacles are an inherent phenomenon in mathematics arising from the need to learn abstraction, notation and interpretation. A further route to deeper understanding about ‘misconceptions’ is to engage teachers in a new area of mathematics and to recognize what happens when their reasoning turns out to be ‘incorrect’.

Why would we think these omissions of perspective are resolved by: studying more maths, or studying special maths?

What do we know about the effects of studying more mathematics on teaching? Mainly what we know is negative: that having higher qualifications does not in itself lead to better teaching that being taught university maths courses as professional development does not necessarily lead to better teaching and can be counter-productive. On the other hand, in developing interactive teaching skills, those who could make most of learners’ ideas were those with richest personal subject knowledge. Take an example presented to a year 9 class:

What knowledge is required to construct such a question? One would need mathematical experience at a more advanced level, both of concepts and of how to combine concepts, than the component parts, plus some understanding of analyzing complex mathematical statements to find familiar structures.

In the discussion of ‘developing and deepening which took place at the September 2007 seminar in this series, indicate that teachers need “deep and transformative subject knowledge of some areas of mathematics”. There is growing interest in the notion of big ideas, key ideas, that provide coherence in the curriculum and this would be one way forward.