# **The Binomial Theorem**

A **binomial** is an algebraic expression containing 2 terms. For example, (*x* + *y*) is a binomial.

## The Binomial Theorem

We use the **binomial theorem** to help us expand binomials to any given power without direct multiplication. As we have seen, multiplication can be time-consuming or even not possible in some cases.

## Properties of the Binomial Expansion (*a* + *b*)*n*

* There are n+1 terms.
* The first term is *an* and the final term is *bn*.
* Progressing from the first term to the last, the exponent of *a* decreases by 1 from term to term while the exponent of *b* increases by 1. In addition, the sum of the exponents of *a* and *b* in each term is *n*.
* If the coefficient of each term is multiplied by the exponent of *a* in that term, and the product is divided by the number of that term, we obtain the coefficient of the next term.

## Binomial Theorem Formula

Based on the binomial properties, the binomial theorem states that the following **binomial formula** is valid for all positive integer values of *n*:

This can be written more simply as:

(a + b)n = nC0an + nC1an − 1b + nC2an − 2b2 + nC3an − 3b3 + ... + nCnbn

We can use the ​n​​C​r​​ button on our calculator to find these values.

This can also be written nCr.

We sometimes need to expand binomials as follows:

(*a* + *b*)0 = 1

(*a* + *b*)1 = *a* + *b*

(*a* + *b*)2 = *a*2 + 2*ab* + *b*2

(*a* + *b*)3 = *a*3 + 3*a*2*b* + 3*ab*2 + *b*3

(*a* + *b*)4 = *a*4 + 4*a*3*b* + 6*a*2*b*2 + 4*ab*3 + *b*4